* Electric charges :- The gravitational force of attraction between two electrons placed 1 cm apart is

$$F_G = \frac{G m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{(10^{-2})^2} N$$

$$= 5.5 \times 10^{-67} N.$$

But an electron is found to repel another electron at 1 cm with a force of .

$$F_E = 2.3 \times 10^{-24} N$$

This extra force is called the electric force. The electric force is very large as compared to the gravitational force. The electron must have some additional property, apart from their mass, which is responsible for the electric force. This additional property is known as charge. Just as masses are responsible for the gravitational force, charges are responsible for the electric force.

** Charges are of two types

(a) Positive charge : lesser number of electrons than number of protons.

(b) Negative charge: More number of electrons than number of protons.

*Note: Only electrons are responsible for a substance to be charged and not the protons.

** Properties of charge:-

(i) Like charges repel while unlike charge attract each other

(ii) Charge is quantized in nature, i.e the magnitude of charge possessed by different objects is always an integral multiple of charge of electron (or proton) i.e the charge of any object

$$Q = \pm ne \qquad n \text{ is an integer.}$$

where $n = 1, 2, 3, 4 \ldots\ldots$

(iii) The minimum possible charge that can exist in nature is the charge of electron which has a magnitude of $e = 1.6 \times 10^{-19} C$. This is also known as quantum of charge or fundamental charge.

(v) The charge of an isolated system is conserved. It is possible to create or destroy charged particles but it is not possible to create or destroy net charge.

* S.I unit of charge is coulomb
* cgs unit of charge is e.s.u. (stat coulomb).

1 coulomb = 3×10^9 esu.

** Coulomb's law :- The force of interaction between any two points are directly proportional to the product of the charges and inversly proportional to the square of the distance between them. The force act along the line joining the two points. If q_1 and q_2 are the two charges seperated by the distance r then the force acting between the two charges is given by

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = K \frac{q_1 q_2}{r^2}$$

where $K = \frac{1}{4\pi \epsilon_0 \epsilon_r}$

$\therefore$ ϵ_r = relative permittivity

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ N m^2 \ coulomb^{-2}$$

$$\therefore F = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$

** Factors affecting coulomb's law :-

* Electric force between two charges not depends upon the neighbouring charges.

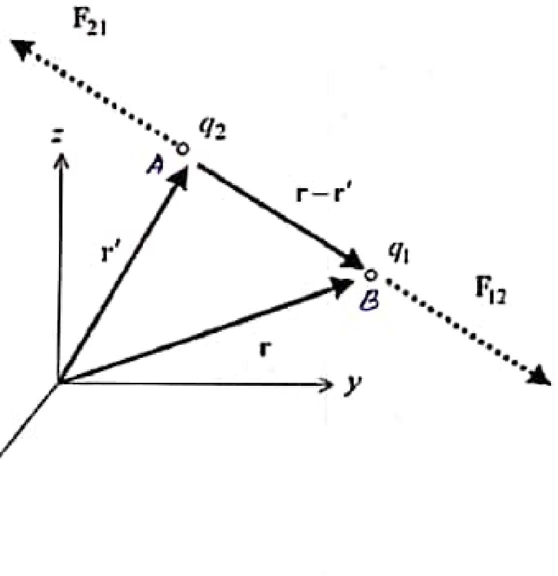
* When two charges (Q_1, Q_2) are placed some distance apart. Neutral point is nearer to the smaller charge and in between Q_1 and Q_2 if charges are like and away from smaller charge if charges are unlike.

* This law is valid only for stationary charges and cannot be applied for moving charges.
* This law is valid only if the distance between two charges is not less than 10^{-15} m.

** Coulomb's law in vector form :–

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}-\vec{r}'|^2}\hat{r}_{21} \longrightarrow (1)$$

$\hat{r}_{21}$ unit vector along the direction $\vec{AB}$.



Now $|\vec{r}-\vec{r}'|\,\hat{r}_{21} = \vec{r}-\vec{r}'$

$\therefore (1) \Rightarrow \vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}-\vec{r}'|^3}\,\vec{r}-\vec{r}'$

$\vec{F}_{12}$ force on charge q_1 due to q_2

Similarly

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}-\vec{r}'|^2}\hat{r}_{12}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}-\vec{r}'|^2}(-\hat{r}_{21}) = -\vec{F}_{12}$$

$\Rightarrow \vec{F}_{21} = -\vec{F}_{12}$

$\vec{F}_{21}$ force on charge q_2 due to q_1.

** Force between multiple charges (Superposition principle):–

Superposition principle states that the total force on any charge due to a number of other charges at rest is the vector sum of all the forces on the charge due to other charges, taken one at a time. The force due to individual charges are unaffected due to the presence of other charges.

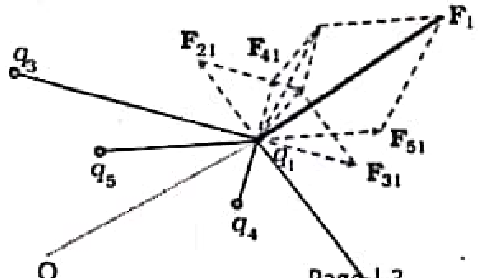
Force on q_1 due to the charge q_2 is $\vec{F}_{12}$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

Similarly, force on q_1 due to q_3 is $\vec{F}_{13}$

$$\vec{F}_{13} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$$

and So on



∴ The net force $\vec{F_1}$ on the charge q_1 due to all other charges is given by

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + \cdots + \vec{F_{1n}}$$

$$\therefore \vec{F_1} = \frac{1}{4\pi\epsilon_0}\left\{ \frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \cdots + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n} \right\}$$

$$\vec{F_1} = \frac{q_1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$

** Dielectric constant :—

Let two charge particles q_1 and q_2 are seperated by a distance 'r' in a medium of permittivity ϵ. Then the force acting between the two charge particles is

$$\vec{F_m} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}\hat{r} \longrightarrow (1)$$

If the same two charge particles are held at the same distance in vacuum then the force between the same particles is

$$\vec{F_0} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \longrightarrow (2)$$

Now

$$(2) \div (1) \Rightarrow \frac{\vec{F_0}}{\vec{F_m}} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$$

ϵ_r is called relative electrical permittivity of the medium or dielectric constant of the medium.

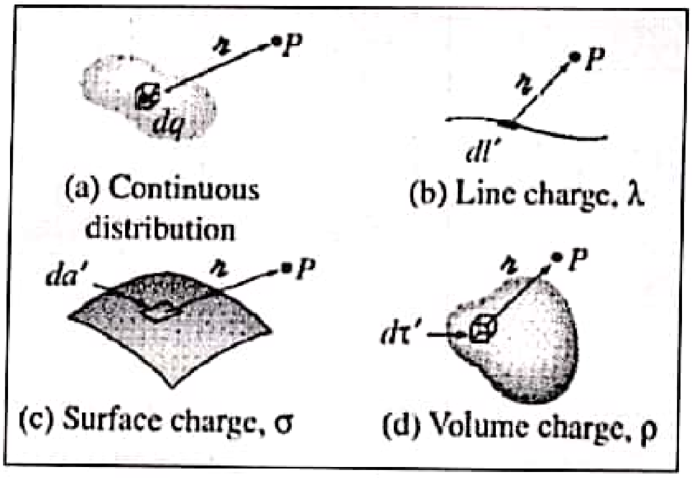
Hence,

Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the absolute permittivity of free space.

Dielectric constant of a medium may also defined as the ratio of force of interaction between two charges seperated by a certain distance in air/vacuum to the force of attraction repulsion between the same two charges, held in the same distance apart in the medium.

Dielectric constant depends only on the nature of medium.

** Continous charge distribution :-



* Linear charge distribution: When the charges are distributed linearly, the total charge dq over a small length dl' is given by

$$dq = \lambda\, dl'$$

where λ is the line charge density. unit of λ : Cm^{-1}

* Surface charge distribution : when the charges are distributed uniformly over a surface, then the total charge dq over the small area da' is given by

$$dq = \sigma\, ds'$$

where σ is the surface charge density, unit of σ is Cm^{-2}.

* Volume charge distribution :- When the charges are distributed uniformly over a volume $d\tau'$, then the total charge over the volume $d\tau'$ is given by

$$dq = \rho\, d\tau'$$

where ρ is the volume charge density, unit of ρ is Cm^{-3}.

** Electric field :- A charge produces something called an electric field in the space around it and this electric field exerts a force on any charge placed on it.

The electric field due to a given charge is the space around the charge in which electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.

* Note :- 1. For measuring $\vec{E}$ practically a test charge (+ve) of magnitude much less than the source charge should be used.

* Note : 2 Electric force on a charge in uniform E is constant and hence acceleration is constant, so equation of motion can be used as

$a = qE/m$

* Electric field intensity:- Force experienced by a unit positive charge placed in an electric field at a point is called the electric field intensity at that point. If q_0 is the test charge placed at any point $(\vec{r})$ and $F(r)$ be the force acting on the charge particle then the electric field intensity at the point is given by

$$\vec{E}(\vec{r}) = \frac{\vec{F}(r)}{q_0}$$

* S.I unit of electric field is Newton / coulomb (N/C).
* Electric field intesity is a vector quantity.

As the test charge $+q_0$ may have its own electric field, it may modify the electric field of the source charge. Therefore to minimise this effect and ultimately remove it, we rewrite electric field intensity at $\vec{r}$ as,

$$\vec{E}(\vec{r}) = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

As the magnitude of the test charge decreases, electric field at a point is defined more and more accurately. On account of the descrete nature of charge, the minimum possible value of test charge q_0 is 1.6×10^{-19} C (which is the unit charge). It cannot be zero.

* Dimensional formula of electric field $[M^1L^1T^{-3}A^{-1}]$

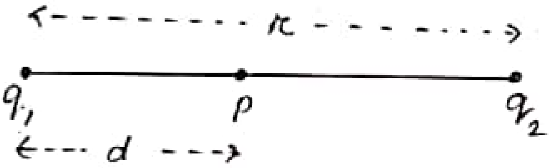
* Direction of electric field due to positive charge is away from charge while direction of electric field due to negative charge is towards the charge.

** Special points to be noted:

a) If q_1 and q_2 are at a distance 'r' and both have the same type of charge, then the distance 'd' of the point from q_1 where electric field is zero is given by

$$d = \frac{\sqrt{q_1}\, r}{\sqrt{q_1} + \sqrt{q_2}}$$

. The point lie between the two charges on the line joining q_1 and q_2.



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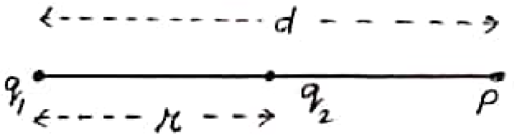
(b) If q_1 and q_2 have opposite charges then the distance 'd' of the point P from q_1 where electric field is zero is given by

$$d = \frac{\sqrt{q_1}\, r}{\sqrt{q_1} - \sqrt{q_2}} \qquad \text{Here } q_1 > q_2$$

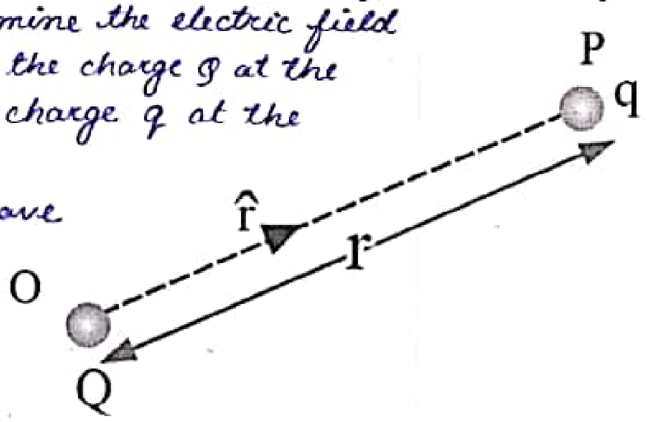


(c) Three charges $+Q_1$, Q_2 and q are placed on a straight line. If these system of charges is in equilibrium, charge q should be as given

$$q = -\frac{Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

** Electric field due to a point charge.

Let us consider a point P at a distance r from the charge Q at point O. We are to determine the electric field intensity at the point P due to the charge Q at the point O. Let us consider a test charge q at the point P.



From Coulomb's law we have the force F between the two point charges is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector directed from the point O to the point P.

Again we know that the electric field

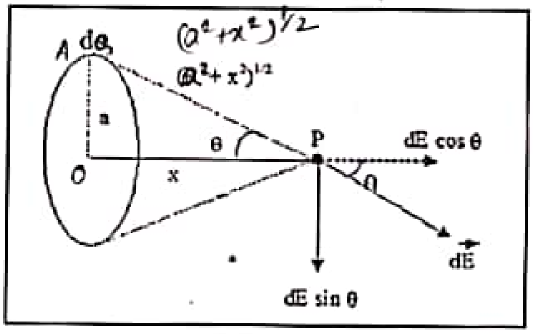
$$\vec{E} = \frac{\vec{F}}{q}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r}|^3} \vec{r}$$

** A ring of radius 'a' contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance of x from the centre of the ring.



Solⁿ:-

Let us consider a small element of the ring at the point A having charge dQ. The field at the point P due to the element is

$$dE = \frac{dQ}{4\pi\epsilon_0 AP^2}$$

Resolving the field $d\vec{E}$ along two rectangular components

$dE \cos\theta$ along the horizontal component

$dE \sin\theta$ along the vertical component.

Here the vertical components of the field i.e $dE\sin\theta$ will cancell out. The net field at the point P is only due to the horizontal component.

∴ Net field at the point P is

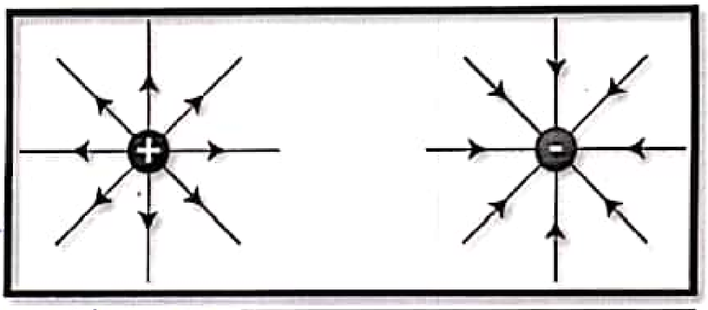
$$\because \cos\theta = \frac{OP}{AP}$$

$$E = \int dE \cos\theta.$$

$$= \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{AP^2} \cos\theta = \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{(a^2+x^2)^{1/2}} \frac{x}{(a^2+x^2)}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x}{(a^2+x^2)^{3/2}} \int_0^Q dQ$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{xQ}{(a^2+x^2)^{3/2}}$$

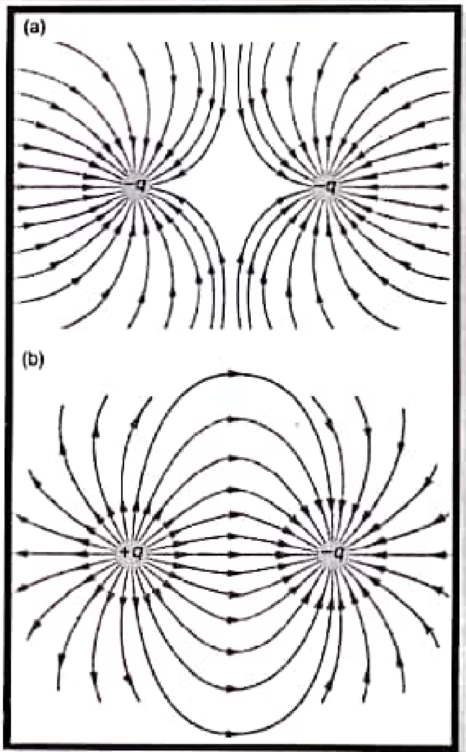


** Electric field lines :-
Electric field line in a region can be graphically represented by drawing certain curves known as lines of electric force or electric field lines.

Lines of force are drawn in such a way that the tangent to a line of force gives the direction of resultant field there.

*. Electric field line due to positive point charge is represented by straight lines originating from the charge

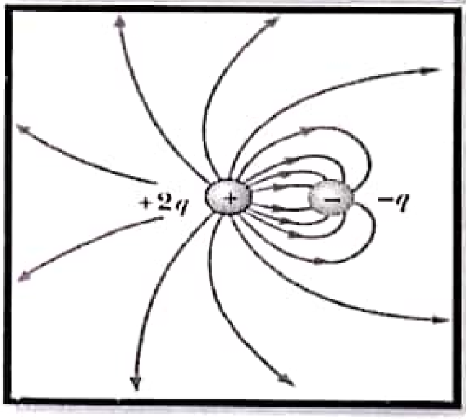
* Electric field line due to negative charge is represented by straight lines termenating at the charge



** Properties of electric field lines :-

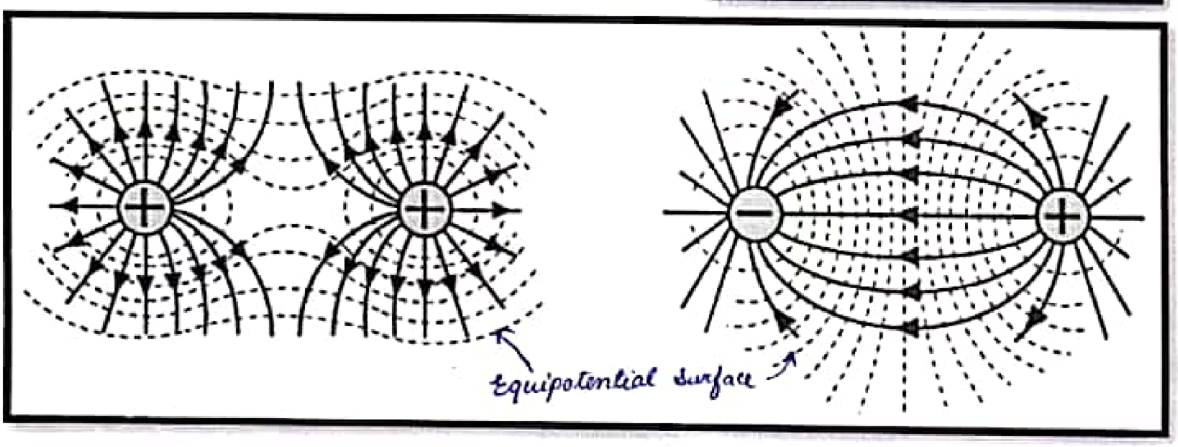
1. Electric lines of force starts from positive charge and end on negative charge.

2. No two lines of force can intersects each other. If they does so Then at the point of intersection two tangent could be drawn, which gives two directions of the field at the same point, which is impossible.



3. Tangent drawn at any point on lines of force gives the direction of the force acting on positive charge placed at that point.

4. These lines have a tandency to contract in length as stretched elastic



string. This actually explains the attraction between opposite charges.

5. These lines have tendency to separate from each other in direction perpendicular to their length. This explains repulsion between like charges.

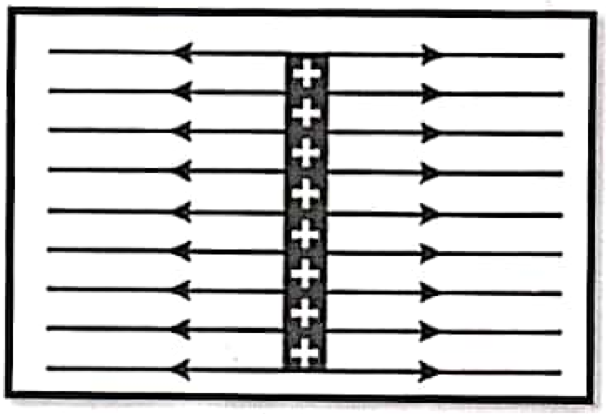
6. Intensity of field line is given by the number of electric field lines in a unit area at that point.

7. Lines of force of a uniform field are parallel and are equally spaced.

8. A unit positive charge gives $\frac{4\pi}{K}$ lines in a medium of dielectric constant K.

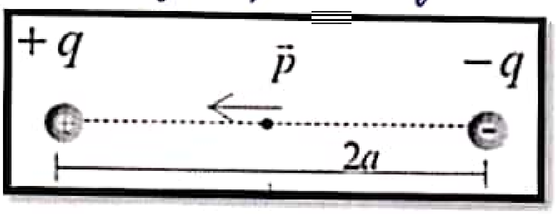
9 Electric lines of force can never enter the conductor, because inside a conductor the intensity of electric field is zero.

10. Lines of force leaves the surface of a conductor normally.



** Electric dipole :-

A system of two equal and opposite charges seperated by a certain distance is called electric dipole, shown in the figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the seperation between the charges.



Dipole moment $\vec{p} = q\vec{d}$ [$d = 2a$ Here]

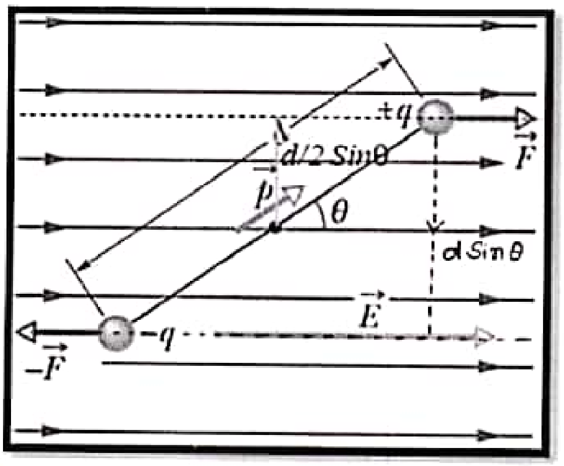
In some molecules the centre of the positive and the negative charges doesn't coincide. This results in formation of dipole. Atom is nonpolar because in it the centre of positive and negative charges concides. Polarity can be in an atom by the application of electric field. Hence it can be called induced dipole.

· Dipole moment is a vector quantity and it is directed from negative to the positive charge.

Dimension of dipole moment $\vec{p} = [L^1 T^1 A^1]$

** Dipole placed in an uniform Electric field :-

Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here charges of the dipole experience forces qE in the opposite direction.



The net force on the dipole

$$\vec{F_N} = (qE - q.E) = 0.$$

Thus when a dipole is placed in an electric field (uniform) net force on the dipole is zero. But equal and opposite forces acts with a seperation of their line of action, they produce a couple which tends to align the dipole along the direction of the field. The torque due to this couple is given by

$$\tau = \text{Force} \times \text{seperation of lines of action}.$$

$$= qE\ d\sin\theta = E\,qd\ \sin\theta = |\vec{E}||\vec{p}|\ \sin\theta$$

$$\Rightarrow \vec{\tau} = \vec{E} \times \vec{p}$$

** Work done in rotating a dipole in Electric field.

Let us consider a dipole of dipole moment $\vec{p}$ is oriented at an angle θ with the direction of the electric field $\vec{E}$. (Show in the figure above). The torque acting on the dipole, $\vec{\tau}$ is given by

$$\tau = pE\sin\theta.$$

This torque tries to rotate the dipole to align it along $\vec{E}$. Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is

$$dW = \tau\, d\theta. = pE\sin\theta\ d\theta.$$

$\therefore$ Total amount of work done in rotating the dipole from orientation θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$= -PE \left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= -PE (\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow W = PE (\cos\theta_1 - \cos\theta_2) \longrightarrow (1)$$

This amount of workdone is equal to the potential energy of the dipole.

$$\therefore U = PE (\cos\theta_1 - \cos\theta_2)$$

* Particular cases:

Case 1:- when the dipole is initially aligned along the electric field. i.e $\theta_1 = 0°$ and we are to set it at angle θ with $\vec{E}$, i.e $\theta_2 = \theta$

$$\therefore W = PE (\cos 0 - \cos\theta)$$

$$= PE (1 - \cos\theta) \qquad [\text{from eq}^n (1)]$$

Case 2:- When the dipole is initially at right angle to $\vec{E}$ ie, $\theta_1 = 90°$, and we have to set it at angle θ with $\vec{E}$ i.e $\theta_2 = \theta$.

$$W = PE (\cos 90° - \cos\theta)$$

$$= -PE \cos\theta$$

** Force on electric dipole in Non-uniform electric field:-

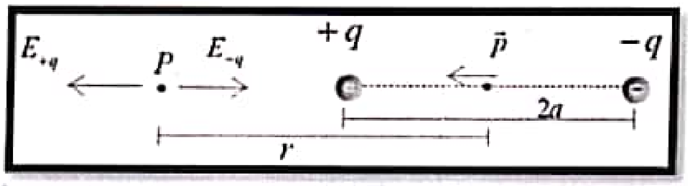
If in a non-uniform electric field a dipole is placed at a point where the field is $\vec{E}$, the interaction energy of dipole at this point is $U = -\vec{P}\cdot\vec{E}$. Now the force on the dipole due to electric field $F = -\frac{\Delta U}{\Delta r}$.

If the dipole is placed in the direction of the field then

$$F = -P \frac{dE}{dx}$$

** Electric field due to a dipole :-

* At a point on the axial line of the dipole :- Let us consider a dipole consisting of two charges $-q$ and $+q$ seperated by a small distance



$2a$. Let us consider a point P at a distance r from the centre of the dipole. If E_{+q} be the electric field at the point P due to the charge $+q$ then $|\vec{E}_{+q}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}$ ..... ①

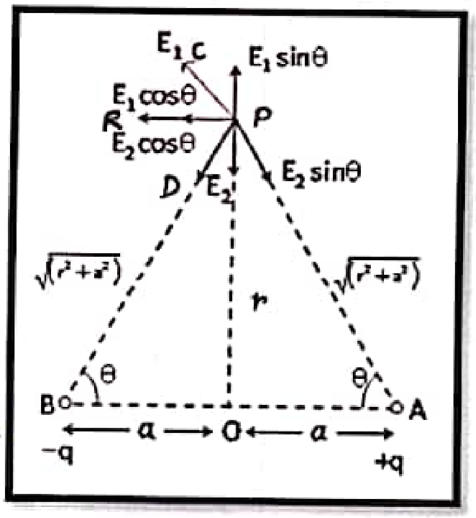
If E_{-q} be the electric field at the point P due to the charge $-q$ then,
$$|\vec{E}_{-q}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \longrightarrow ②$$

$\therefore$ Net electric field at the point P is
$$|\vec{E}|^2 = |\vec{E}_{+q}|^2 + |E_{-q}|^2 + 2E_{+q}E_{-q}\cos 180^\circ$$
$$\Rightarrow |\vec{E}| = |\vec{E}_{+q}| - |\vec{E}_{-q}| = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right\}$$
$$\Rightarrow |\vec{E}| = \frac{q}{4\pi\epsilon_0}\frac{4ar}{(r^2-a^2)^2} = \frac{1}{4\pi\epsilon_0}\frac{2r|\vec{p}|}{(r^2-a^2)^2}$$

For a shorter dipole $2a << r$ then
$$\vec{E}_{axial} = \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \qquad \because r^2-a^2 \approx r^2$$

** At a point on the equatorial line of the dipole :-

Let us consider an electric dipole consisting of two charges $-q$ and $+q$ seperated by a small distance $2a$ with centre at O.



The dipole moment $\vec{p} = q(2a)$

We are to find the electric field intensity at the point P on the equatorial line at a distance of r from the centre of the dipole.

Let E_1 be the field at the point P due to the charge $+q$ at A.
$$\therefore |\vec{E}_1| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \longrightarrow ① \text{ (along } \vec{PC})$$

Similarly electric field at the point P due to the charge $-q$ at the point B

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \longrightarrow ② \quad (\text{along } \vec{PD})$$

$$①, ② \Rightarrow |\vec{E_1}| = |\vec{E_2}| \longrightarrow ③$$

Now resolving the two fields $\vec{E_1}$ and $\vec{E_2}$ along the two rectangular components as shown in the figure.

The two verticle components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and are acting along the opposite direction. Hence they cancel out.

Therefore the net electric field at the point P is only due to the horizontal component. The net electric field at the point P is $\therefore |\vec{E}| = |\vec{E_1}|\cos\theta + |\vec{E_2}|\cos\theta$

$$= 2E_1\cos\theta \qquad \because |\vec{E_1}| = |\vec{E_2}|$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}\cos\theta.$$

$$= \frac{2qa}{4\pi\epsilon_0(r^2+a^2)^{3/2}} \qquad \because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{-\vec{p}}{(r^2+a^2)^{3/2}} \longrightarrow ④$$

The direction of $\vec{E}$ is along $\vec{PR}$ and $\vec{p}$ is opposite to $\vec{E}$.

For a shorter dipole $r >> a \Rightarrow r^2+a^2 \approx r^2$

$$\therefore ④ \Rightarrow |\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

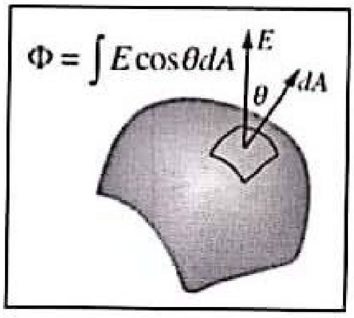
$$\Rightarrow \vec{E}_{equ} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

Note

$$\frac{|\vec{E}_{axial}|}{|\vec{E}_{equ}|} = 2$$

** Electric flux:- It is defined as the total number of lines of force passing normally through any surface placed in the field. It is denoted by ϕ and is a scalar quantity.

It is given by the dot product of $\vec{E}$ and normal infinitesimal area $\vec{dA}$ integrated over a closed surface



$$d\phi = \vec{E}\cdot\vec{dA} = E\,dA\cos\theta$$

$$\Rightarrow \phi = \int E\cos\theta\, dA$$

where θ is the angle between the electric field and normal to the area.

*Note:-(i) SI unit of flux V-m and Nm^2/C

(ii) Dimension: $[ML^3T^{-3}A^{-1}]$

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

(iv) Flux due to the positive charge goes out of the surface while that due to negative charge comes into the surface.

(v) Flux entering a surface is taken as positive while flux leaving is taken as negative.

(vi) The value of electric flux is independent of the shape and size of the closed surface.

(vii) If only a dipole is present within a closed surface then net flux is zero.

(viii) Net flux associated with a closed surface kept in a uniform electric field is zero.

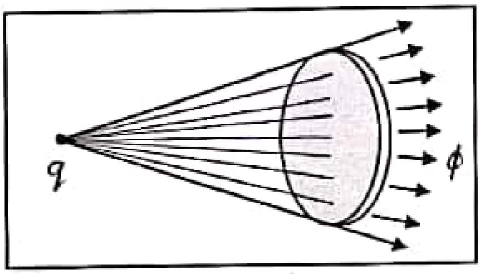
(ix) Net flux from a surface is zero does not imply that intensity of field is also zero.

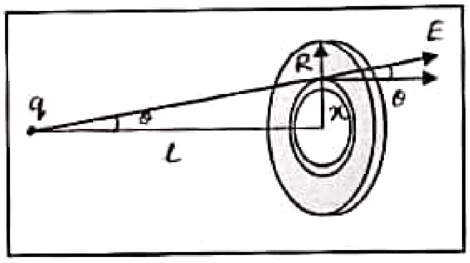
*** (For JEE)

*Electric flux through a circular disc due to a point charge:-

Let us consider a point charge q at a distance l from a disc of radius R. We are to find the electric flux through the disc surface due to the point charge q. The point charge q originates electric field lines radially outward direction. The field lines originated in a cone that passes through the disc surface.

To calculate the flux, let us consider an elemental ring on the disc surface of radius x and width dx as shown in the figure. Area of the ring strip is $dS = 2\pi x\, dx$. The electric field due to the point charge q at this elemental ring is given by $$E = \frac{1}{4\pi\epsilon_0}\frac{q}{(x^2+l^2)} = K\frac{q}{(x^2+l^2)} \longrightarrow ①$$





If $d\phi$ is the flux passing through this elemental ring, then

$$d\phi = E\,dS\cos\theta = \frac{Kq}{(x^2+l^2)}\,2\pi x\,dx\,\frac{l}{\sqrt{x^2+l^2}}$$

$$= \frac{2\pi K q\,l\,x\,dx}{(x^2+l^2)^{3/2}}$$

Total flux

$$\Rightarrow \phi = \int d\phi = \int_0^R \frac{q\,l\,\cancel{2\pi}}{_2\cancel{4\pi}\epsilon_0}\frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \int_0^R \frac{ql}{2\epsilon_0}\frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \frac{ql}{2\epsilon_0}\int_0^R \frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \frac{ql}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+l^2}}\right]_0^R$$

$$\Rightarrow \phi = \frac{ql}{2\epsilon_0}\left[\frac{1}{l} - \frac{1}{\sqrt{R^2+l^2}}\right]$$

*** **Gauss law :-** This law states that the electric flux ϕ_E through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

i.e $$\phi_E = \oint \vec{E}\cdot d\vec{S}$$

$$= \frac{q_{enc}}{\epsilon_0}$$

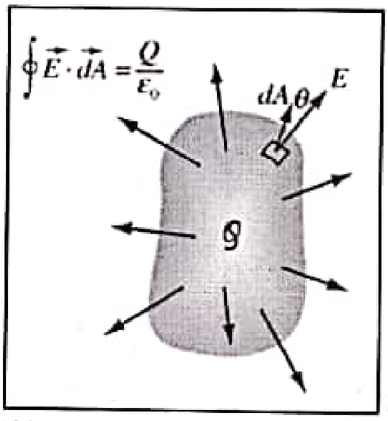
Here the charge enclosed by the surface $q_{enc} = Q$.

The closed surface can be hypothetical and then it is called a Gaussian surface.

If the closed surface enclosed a number of charges $q_1, q_2 \cdots\cdots q_n$, then

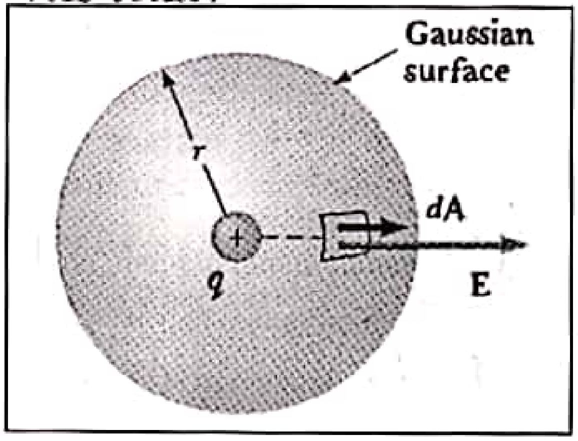
$$\phi = \int \vec{E}\cdot\vec{ds}$$

$$= \frac{(q_1 + q_2 + \cdots\cdots + q_n)}{\epsilon_0}$$



** Gauss's law and Coulomb's law are equivalent, i.e if we assume Coulomb's law we can prove Gauss's law and vice-versa.

To prove Gauss's law from coulomb's law let us consider a gaussian surface of radius r with point charge q at the centre as shown in the figure.



From coulomb's law the electric field intensity at the point P on the surface will be

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q\vec{r}}{|\vec{r}|^3}$$

And the electric flux linked with the area $d\vec{A}$ is

$$d\phi = \vec{E}\cdot d\vec{A} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\vec{r}\cdot\vec{ds}$$

$\therefore$ The total flux linked with the whole spherical surface

$$\phi = \int d\phi = \int \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}|^3}\vec{r}\cdot\vec{ds}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\int \hat{r}\cdot\vec{ds}$$

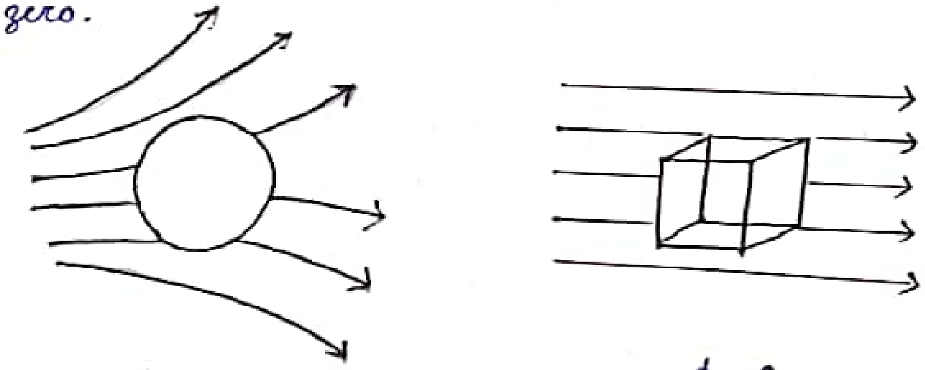
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\int |\hat{r}|\,|\vec{ds}|\cos 0^\circ$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{\cancel{r^2}}4\pi\cancel{r^2}$$

$$\phi = \frac{q}{\epsilon_0}$$

Though here we have assumed the surface to be spherical, it is true for any arbitrary closed surface.

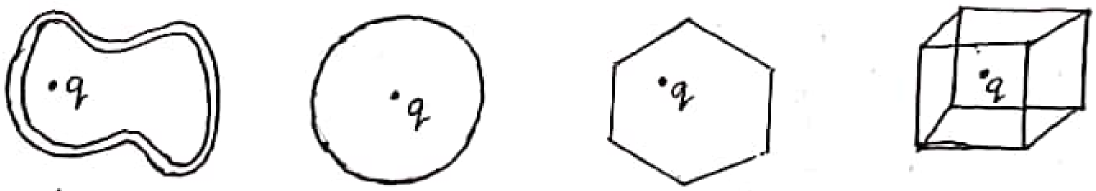
* Note:- If a body (not enclosing any charge) is placed in an electric field either uniform or non uniform total flux linked with the body is will be zero.



$\phi = 0$ $\phi = 0$

*Note:- If the closed body encloses a charge q, then total flux, linked with the body will be

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$



$\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$

From the above expression of flux and the figure it is clear that the flux linked with a closed body is independent of the shape and the size of the body and the position of the charge inside the body.

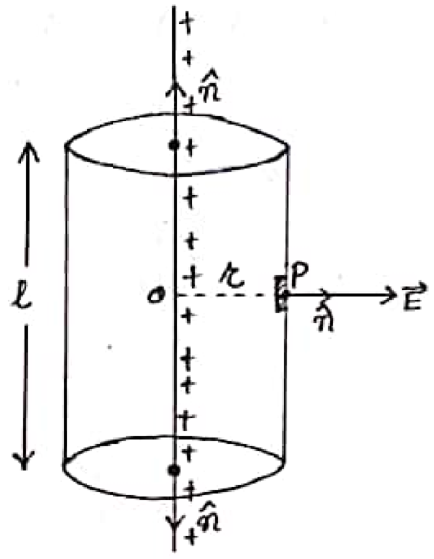
* Note:- For the case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}\phi_E = \frac{q}{2\epsilon_0}$ and if the symmetrical closed body has 'n' identical faces with point charge at its centre flux linked with each face will be $\frac{\phi}{n} = \frac{q}{n\epsilon_0}$.

* Note: Gauss law is a powerful tool for calculating electric field intensity in case of symmetrical charge distribution by choosing a Gaussian surface in such a way that $\vec{E}$ is parallel or perpendicular to its various faces.

** Application of Gauss's law :-

*1. Electric field due to an infinitely long straight uniformly charged wire :-

Let us consider an infinitely long thin wire of uniform linear charge density λ. We are to calculate the electric field intensity at any point P at a distance r from the wire.



Let us consider a cylindrical Gaussian Surface of radius r and length 'l' as shown in the figure. The magnitude of the electric field at each point on the curved surface of the cylinder is same. Let the field at the point P be E.

Here for the curved surface the angle between the field $\vec{E}$ and the area ($\hat{n}$) is $0°$ and for the two plane surface of the cylinder the value of angle is $90°$. Hence only the curved surface contribute to the electric flux.

$\therefore$ Electric flux through the Gaussian surface is

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \int_{curved} \vec{E}\cdot\vec{ds} + \int_{Plane} \vec{E}\cdot\vec{ds} + \int_{Plane} \vec{E}\cdot\vec{ds}$$

$$= \int_{Curved} E\, ds \cos 0° + 2\int_{Plane} E\, ds \cos 90°$$

$$\phi = \int_{Curved} E\, ds = E\int_0^{2\pi r l} ds = 2E\pi r l \longrightarrow ①$$

From the Gauss law we have

$$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \longrightarrow ②$$

$$①, ② \Rightarrow 2E\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\epsilon_0}$$

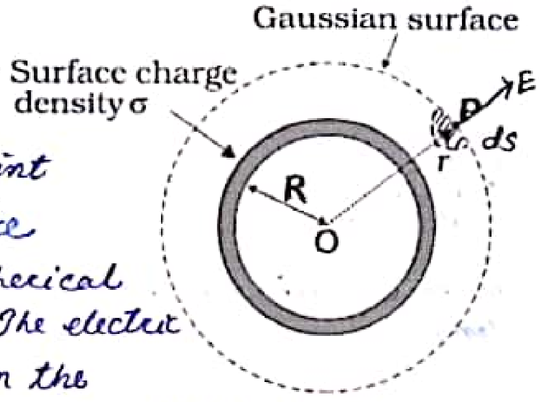
$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow E \propto \frac{1}{r}$$

*2. Electric field due to uniformly charged spherical shell :-

(a) At a point outside the shell.

Let us consider a spherical shell of radius R with centre at O. Let a charge +q be distributed uniformly over the surface of the shell. We are to determine the electric field intensity at any point P at a distance r from the centre of the shell. Let us consider a spherical Gaussian surface of radius 'r'. The electric field intensity at every points on the Gaussian surface is same and say this field be E.



From Gauss's law we have the electric flux through the Gaussian surface

$$\phi = \oint \vec{E}\cdot\vec{ds} = \frac{q_{enc}}{\epsilon_0}$$ Here $q_{enc} = q = \sigma 4\pi R^2$

$$\Rightarrow \oint |\vec{E}||\vec{ds}| \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \oint ds = \frac{q}{\epsilon_0}$$

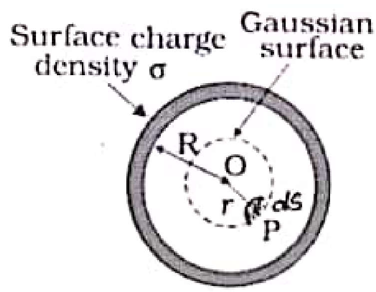
$$\Rightarrow E\, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$ The direction of the field E is radially outward.

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{\sigma 4\pi R^2}{r^2} \longrightarrow ①$$

(b) At a point inside the shell:-

Let us consider a spherical Gaussian surface through the point P of radius r. We are to determine the electric field intensity at the point P. Here the point P is inside the shell ($\because r < R$)



$\therefore$ From Gauss law,

$$\phi = \oint E\cdot ds = \frac{q_{enc}}{\epsilon_0}$$

Here $q_{enc} = 0$

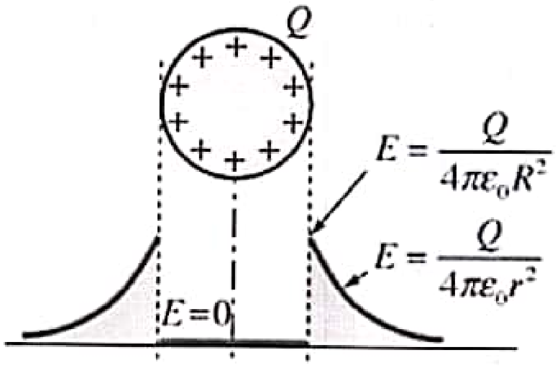
$$\therefore E = 0 \longrightarrow ②$$

c. At the surface of the spherical shell.

At the surface of the sphere $r = R$

$\therefore$ From eq^n (1)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r} \longrightarrow (3)$$



$\therefore$ From eq^n (1), (2) and (3)

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad r > R$$

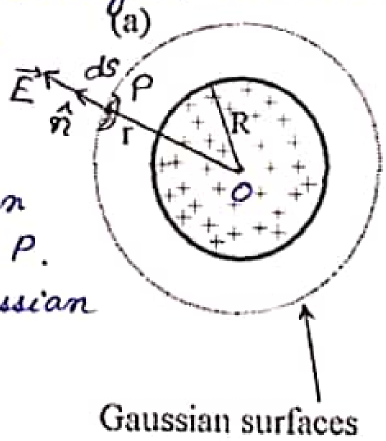
$$E_{in} = 0 \qquad r < R$$

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} \qquad r = R \quad (\text{Maximum value of } E)$$

* 3. Electric field due to a non-conducting charged solid Sphere:-

a. At a point outside the sphere

Let us consider a non-conducting solid sphere of radius R and centre at O. has uniform charge density ρ. We are to calculate the electric field intensity at a point P outside the sphere at a distance 'r' from the centre of the sphere.



Let us consider a spherical Gaussian surface of radius 'r' through the point P. Electric field at each point of the Gaussian surface is same and say it is E.

$\therefore$ From Gauss's law

$$\phi = \oint E.ds = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\oint ds = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E\,4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \rightarrow (1)$$

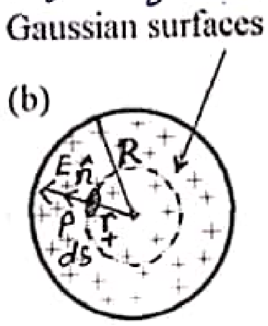
Here angle between $\vec{E}$ and the area $ds\hat{n}$ is 0°. and Q is the total charge enclosed by the Gaussian surface. $Q = \rho\frac{4}{3}\pi R^3$.

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\rho\,\frac{4}{3}\pi R^3}{r^2}\hat{r}$$

Hence $\vec{E}_{out} \propto \frac{1}{r^2}$

(b). At a point inside the sphere:

Let us consider a spherical Gaussian surface of radius r (such that $r<R$) through the point P. We are to find the electric field at the point P.



From the Gauss law we have

$$\phi = \oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

Here the electric field E is same at every point on the Gaussian surface. The angle between the field $\vec{E}$ and the area $ds\hat{n}$ is $0°$.

$$\therefore \int E\, ds \cos 0° = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\int ds = \frac{4}{3}\frac{\pi r^3 \rho}{\epsilon_0}$$

$$\Rightarrow E\, 4\pi r^2 = \frac{4}{3}\frac{\pi r^3\rho}{\epsilon_0}$$

$$\Rightarrow E = \frac{4}{3}\frac{1}{4\pi\epsilon_0}\frac{\pi r^3 \rho}{r^2}$$

$$\Rightarrow \vec{E} = \frac{r\rho}{3\epsilon_0}\hat{r} \longrightarrow ②$$

$$\Rightarrow E_{in} \propto r$$

Here

$$q_{enc} = \frac{Q}{4/3\,\pi R^3}\times \frac{4}{3}\pi r^3$$

where

$$\rho = \frac{Q}{4/3\,\pi R^3}$$

$$\therefore q_{enc} = \rho\,\frac{4}{3}\pi r^3$$

(c) At a point on the surface of the sphere

Here $r = R$.

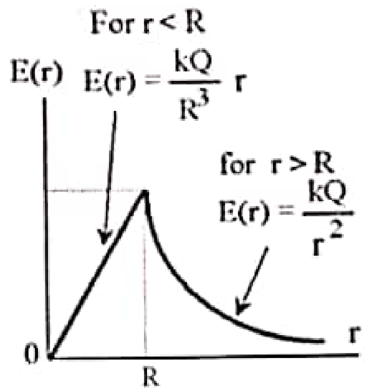
$$\therefore E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r} \longrightarrow ③$$

From eq^n ①, ② and ③ we have

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad r>R$$

$$E_{in} = \frac{1}{4\pi\epsilon_0}\frac{4}{3}\pi r\rho = \frac{r\rho}{3\epsilon_0}\hat{r} \qquad r<R$$

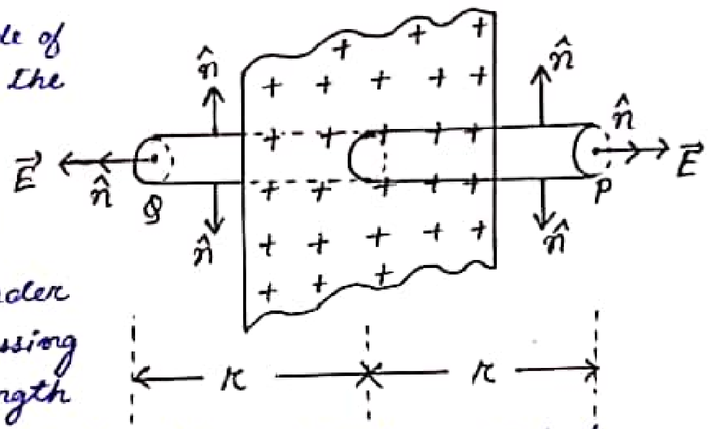
$$E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r}.$$



* 4. Electric field due to a thin infinite plane sheet of charge:-

Let us consider a thin, infinite plane sheet of charge having charge density σ (surface charge density). we are to determine the electric field intensity at a point P. at a distance r from the sheet.

Electric field on either side of the sheet is perpendicular to the sheet and have the same magnitude at all points equidistant from the sheet.



Let us imagine a cylinder of cross-sectional area ds passing through the sheet having length $2r$. At the two edges of the cylinder the outward normal to the surface is parallel to the field E.

$\therefore$ The electric flux through these edges $= 2\vec{E}.dS\hat{n}$
$= 2EdS.$

The curved surface of the cylinder will not contribute to the electric flux, since for the curved surface the outward normal $\hat{n}$ is perpendicular to the field E.

$\therefore$ The total flux through the cylindrical Gaussian surface is

$$\phi = 2EdS.$$

Now from the Gauss's law we have

$$\phi = 2EdS = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow 2EdS = \frac{\sigma dS}{\epsilon_0}$$

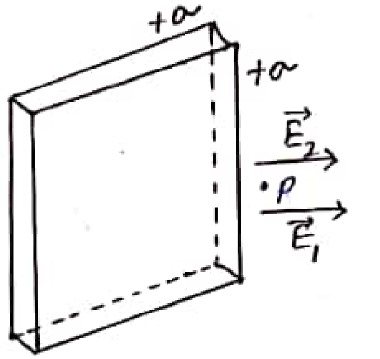
$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Here we have seen that the field E is independent of the distance r.

* Special case:- If the infinite plane sheet has uniform thickness and have uniform surface charge density σ on both the surface of the sheet.

The electric field intensity at any point P due to each surface is $E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$

From the superposition principle we have the net electric field intensity at the point P due to the infinite plane sheet



$$E = E_1 + E_2$$
$$= \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$
$$= \frac{\sigma}{\epsilon_0}$$

* Electric field intensity due to two thin infinite parallel sheet of charge.

Let two thin infinite plane sheets of charge density σ_1 and σ_2 are held parallel to each other.

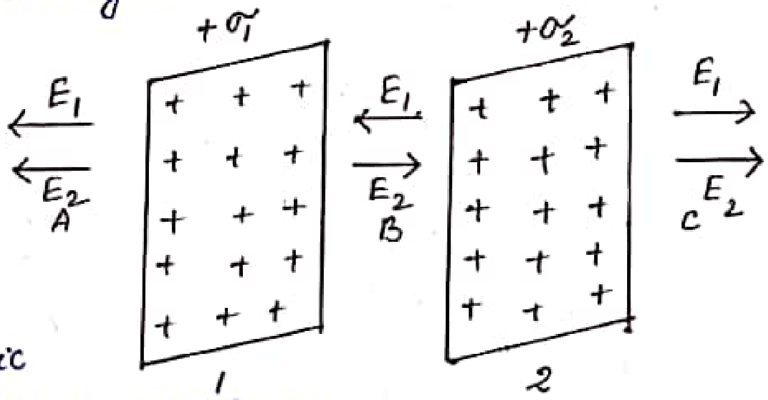
Let E_1 and E_2 are the electric field intensities at a point due to the charged sheet 1 and 2.

Then we have

$$E_1 = \frac{\sigma_1}{2\epsilon_0}$$

and

$$E_2 = \frac{\sigma_2}{2\epsilon_0}$$



From the figure electric field in the region A, B and C are

$$E_A = E_1 + E_2 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

$$E_B = E_1 - E_2 = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$$

$$E_C = E_1 + E_2 = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

Special case: If $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$

Then

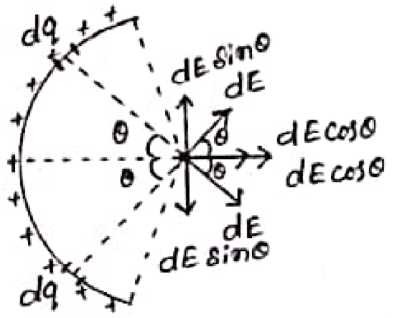
$$E_A = \frac{1}{2\epsilon_0}(\sigma - \sigma) = 0$$

$$E_B = \frac{1}{2\epsilon_0}(\sigma + \sigma) = \frac{\sigma}{\epsilon_0}$$

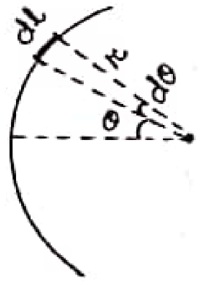
$$E_C = \frac{1}{2\epsilon_0}(\sigma - \sigma) = 0.$$

* Problem :- Linear charge density of an arc is λ and the total angle at the centre is α. Calculate the electric field at the centre.

Let us consider two small elements of the arc of length dl having charge dq as shown in the figure.



If dE be the field at the centre due to the two elements then resolving the two fields as shown in the figure.



From figure we have the net field due to the two elements is

$$dE_N = dE\cos\theta + dE\cos\theta$$
$$= 2\,dE\cos\theta.$$

Again
$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dl}{r^2}$$

$$\therefore dE_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda\,dl}{r^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{2\lambda(r\,d\theta)}{r^2}\cos\theta.$$

$$\because dq = \lambda\,dl\,,\ dl = r\,d\theta$$

$$\Rightarrow dE_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\cos\theta\,d\theta$$

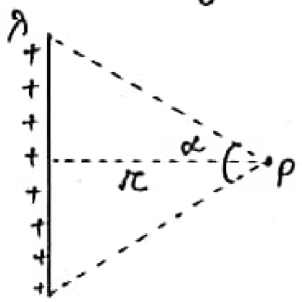
$\therefore$ Total field at the centre is

$$E_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\int_0^{\alpha/2}\cos\theta\,d\theta = \frac{2K\lambda}{r}\sin(\alpha/2)$$

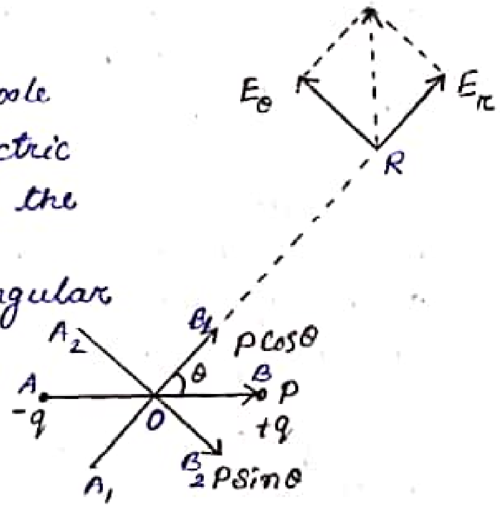
$$\Rightarrow E_N = \frac{2K\lambda}{r}\sin\left(\frac{\alpha}{2}\right)$$

* Note: Electric field due to linear charge distribution :-

$$E_P = \frac{2K\lambda}{r}\sin(\alpha/2)$$



* Electric field due to dipole :-



Let us consider a electric dipole of dipole moment $\vec{P}$. we are to determine the electric field intensity at any point R due to the dipole.

Let us resolve $\vec{P}$ into two rectangular components $p\cos\theta$ and $p\sin\theta$ as shown in the figure.

Electric field intensity at the point R on the axial line of A_1B_1,

$$|\vec{E_r}| = \frac{2(p\cos\theta)}{4\pi\epsilon_0 r^3} \longrightarrow (1)$$

Again, Electric field intensity at the point R on the equitorial line of A_2B_2 is

$$|\vec{E_\theta}| = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \longrightarrow (2)$$

$\therefore$ Net electric field at the point R due to the dipole is,

$$|\vec{E}| = \sqrt{E_r^2 + E_\theta^2 + 2E_r E_\theta \cos 90^\circ}$$

$$= \left[\left\{\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right\}^2 + \left\{\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right\}^2\right]^{1/2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

$$|\vec{E}| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\epsilon_0 r^3} \longrightarrow (3)$$

Special cases :- when $\theta = 0^\circ$ then

$(3) \Rightarrow |\vec{E}| = \frac{2p}{4\pi\epsilon_0 r^3}$ (Field due to dipole on the axial line)

Special case :- when $\theta = 90^\circ$ then

$(3) \Rightarrow |\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}$ (Field due to dipole on the equitorial line)