

# SOLOW'S MODEL OF GROWTH

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*Presented by:-*

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## • INTRODUCTION

R.M. Solow has a very prominent place in the galaxy of the neo-classical economists. He developed a steady state growth model involving a variable capital-output ratio and recognized the impact of changes in the distributive shares of labour and capital. He worked his model of growth considering the technical progress as neutral. He emphasized that the economic system, over a long time, will not traverse a knife-edge equilibrium growth path, as had been discussed by Harrod-Domar and Hicks.

## • ASSUMPTIONS

Solow's model of growth is a variant of neoclassical theory in the absence of technical progress. It rests upon the following assumptions:

- Single-commodity and single-sector system is in existence.
- There is fully employed, closed, laissez faire economy.
- Production is governed by constant returns to scale.
- Two factors of production, capital and labour are substitutable.
- Each factor is paid a price equal to its marginal physical product.
- There are perfectly competitive market conditions.
- Saving co-efficient is constant.
- There is neutral technical progress.
- There is perfect flexibility of wage, prices and interest rates.

- **SOLOW'S FUNDAMENTAL GROWTH EQUATION**

Solow gives the following basic equation :

$$K = sF(K, L) \quad \dots(i)$$

Here  $K$  is the rate of increase of capital stock,  $s$  is the fraction of output that is saved.  $F(K, L)$  is the production function which specifies that output is function of capital ( $K$ ) and labour ( $L$ ). The labour force in any time period is determined as

$$L(t) = L_0 e^{nt}$$

Here  $t$  is time and  $n$  is the rate at which labour grows. Solow considers  $n$  as equivalent to Harrod's natural rate of growth in the absence of technological progress.

The equation (i) specifies the rate of accumulation of capital stock which is constant with the growth of labour force.

To demonstrate that there is always a capital accumulation path consistent with any rate of growth of labour force, R.M. Solow introduces the following equation:

$$r = sF(r, 1) - nr \quad \dots (ii)$$

In the above equation,  $r$  is the ratio of capital to labour ( $K/L$ ),  $\dot{r}$  is the rate of increase of capital-labour ratio per unit of time. The equation (ii) can be derived as below:

$$\frac{\dot{r}}{r} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \quad \dots(iii)$$

Here the relative rate of change of  $r$  is equal to difference between the relative rates of  $K$  and  $L$ .

$$\text{Now } \frac{\dot{L}}{L} = n \text{ and } \dot{K} = sF(K, L)$$

Substituting these relations in equation (iii)

$$\frac{\dot{r}}{r} = \frac{sF(K, L)}{K} - n$$

Multiplying both sides by  $r$

$$\dot{r} = \frac{rsF(K, L)}{K} - nr$$

$$\dot{r} = rsF(1, L/K) - nr$$

$$\dot{r} = rsF(1, 1/r) - nr \quad \left[ \because \frac{L}{K} = \frac{1}{r} \right]$$

$$\dot{r} = sF(r, 1) - nr$$

On the basis of this fundamental equation, Solow analysed the *possible growth patterns*.

- **POSSIBLE GROWTH PATTERNS**

The line passing through origin in Fig. 1 with slope  $n$  represents function  $nr$ . The other curve represents the function,  $sF(r, 1)$ . It is so drawn as to show the decreasing marginal productivity of capital. At the point of intersection of two curves  $nr = sF(r, 1)$  and  $r = 0$ . In this situation,  $r = r'$ . In the figure, the intersection between  $sF(r, 1)$  and  $nr$  determines  $r$ , capital-labour ratio. In case this ratio is established, “it will be maintained and capital and labour will grow thence-forward in proportion. Given constant returns to scale, real output will also grow at the same relative rate  $n$ , and output per head of labour force will also be constant.

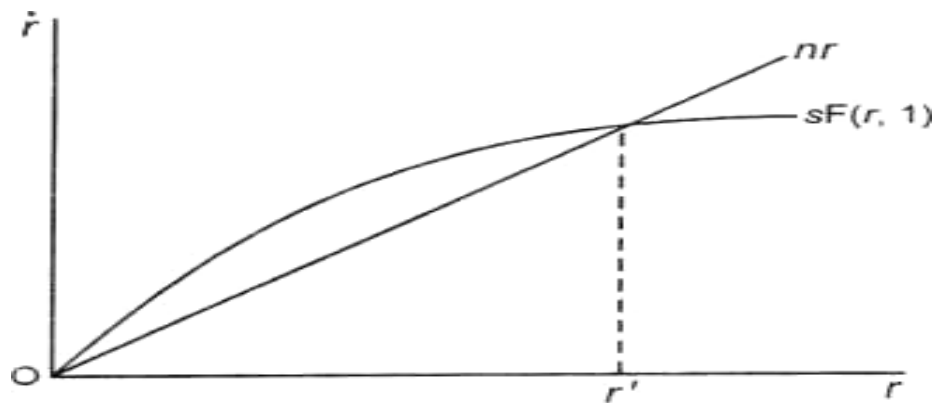


Fig. 1

If  $r > r'$  then  $nr > sF(r, 1)$  and  $r$  will tend to fall so as to approach  $r'$ . On the contrary, if  $r < r'$ , then  $nr < sF(r, 1)$  and  $r$  will tend to increase upto  $r'$ . This shows that the equilibrium value  $r'$  is stable.

- **Stable and Unstable Growth Patterns**

The stability solution given above is not inevitable. It depends upon the shape of productivity of capital curve  $sF(r, 1)$ . The *unstable and stable growth patterns* can be explained through Fig. 2.

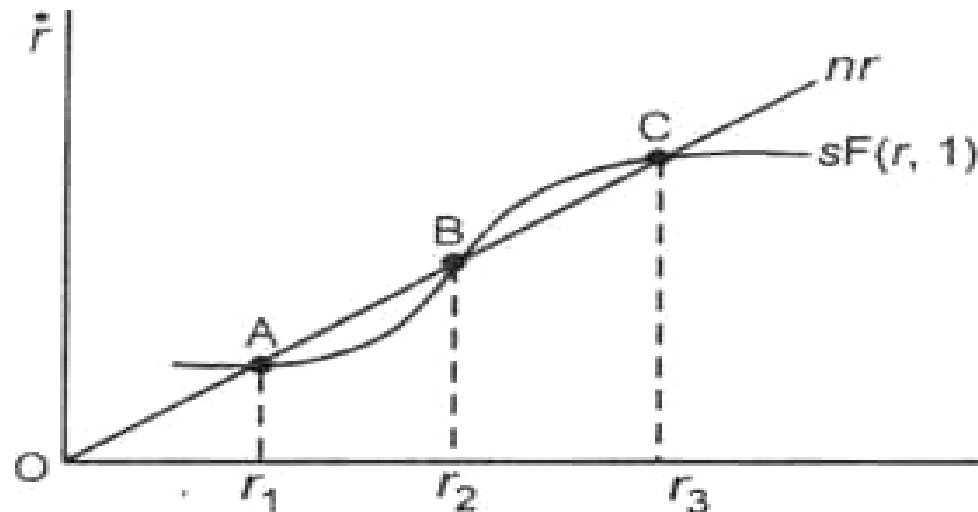


Fig. 2

The productivity of capital curve  $sF(r, 1)$  in Fig. 2 intersects the ray  $nr$  at A, B and C where the equilibrium capital-labour ratios are  $r_1$ ,  $r_2$  and  $r_3$  respectively. While A and C are the *stable* equilibrium positions, B is an *unstable* equilibrium position. If capital-labour ratio is slightly less than  $r_2$ ,  $nr > sF(r, 1)$  so that  $r$  will tend to fall and approximate to  $r_1$  at equilibrium position A. If  $r$  is less than  $r_1$  then  $nr < sF(r, 1)$  so that  $r$  will approximate to  $r_1$ . Thus  $r_1$  is the stable equilibrium capital-labour ratio corresponding to low equilibrium income or output. If there is any accidental disturbance from  $r_2$ ,  $sF(r, 1) > nr$  and the capital-labour ratio will rise and finally approximate to  $r_3$ . In case the capital-labour ratio is more than  $r_3$ , then  $nr > sF(r, 1)$  and consequently capital-labour ratio will fall back to  $r_3$ . Thus  $r_3$  is the stable equilibrium capital-labour ratio. At C there is stable balanced growth equilibrium corresponding to high level of income or output.



- Highly Productive and Highly Unproductive Growth Patterns

Apart from the growth patterns described through Figs. 1 and 2, Solow discussed two more growth patterns through Fig. 3

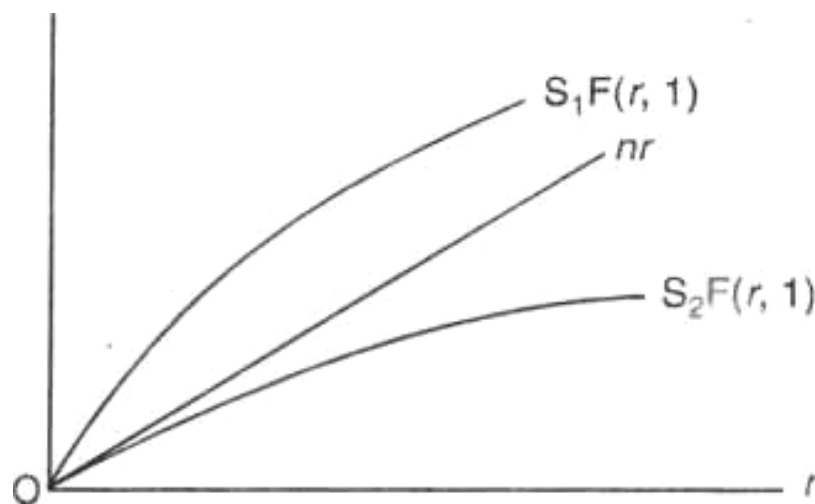


Fig. 3

Suppose the equilibrium growth path is  $nr$ . Along it, warranted and natural growth rates are equal. The growth path of capital is  $s_1 F(r, 1)$  which throughout lies above  $nr$ . Such a situation represents a *highly productive system*. The growth rate of capital remains higher than that of labour so that there is perpetual rise in capital-labour ratio. There is continuous full employment. Income in absolute terms and per head also goes on increasing. If the growth path of capital is  $s_2 F(r, 1)$  which throughout lies below  $nr$ , the economic system is said to be *highly unproductive*. Since growth rate of capital is less than that of labour, the capital-labour ratio continues to fall. Since net investment is always positive, the income in absolute terms increases. The income per head however, goes on diminishing.

This provides the basis for Solow's repudiation of knife-edge equilibrium. In his words, "When production takes place under the usual neo-classical conditions of variable proportions and constant returns to scale, no simple opposition between natural and warranted rates of growth is possible. There may not be ...any knife edge. The system can adjust to any given rate of growth of the labour force and eventually approach a state of steady proportional expansion."

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THANKS

