* Integral Surface passes through a given Curve:

The method:

Let $Pp + Qq = R$ be a given Quasi-linear equation. Let

$$u(x, y, z) = C_1, \quad v(x, y, z) = C_2 \rightarrow (2)$$

be the solutions of the auxiliary equation for the given equation.

Suppose we want to find the integral surface of the given equation that passes through the curve whose parametric equation is given by-

$$x = x(t), \quad y = y(t), \text{ and } z = z(t). \rightarrow (3)$$

Where t is a parameter.

using (3), (2) can be expressed as-

$$u[x(t), y(t), z(t)] = C_1, \quad v[x(t), y(t), z(t)] = C_2 \rightarrow (4)$$

We then eliminate t from the equations in (4) and establish a relation between C_1 and C_2 and replacing C_1 and C_2 using (2) we will get the integral surface of the given equation that passes through the given curve.

* ALGORITHM: $(Pp+Qq=R,\ x=x(t),\ y=y(t),\ z=z(t))$

step I: Find the solution of the auxillary equation of the given equation.

step II: Replace x, y, z, by $x(t), y(t), z(t)$ respectively in u and v, obtained in step I.

step III: Eliminate t from the given equations. and find a relation between C_1 and C_2.

step IV: Eliminate C_1 and C_2 from step I, and step IV to obtain the integral surface.

* Conversion of Certesian equation to parametric equation:

Ex1: Consider the curve $x+y=3$, $z=5$

then by putting $x=t$, we get

$y=3-t$.

$\therefore$ The parametric equation of the curve is—

$x=t$, $y=3-t$, and $z=5$.

Ex2: $x^2+y=1$, $z=t^3$.

putting $x=t$, we have—

$y=1-t^2$

$\therefore$ The parametric equation is—

$x=t$, $y=1-t^2$, and $z=t^3$.

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Ex: Find the integral surface of the equation $xp + yq = z$, passing through the curve $x + y = 1$, $z = t$.

Solution: Given equations is—

$$xp + yq = z \rightarrow (1)$$

using Lagrange's method and solving for the auxiliary equation we have—

$$\left.\begin{aligned} u(x,y,z) &= \frac{x}{y} = c_1 \text{ and} \\ v(x,y,z) &= \frac{y}{z} = c_2 \end{aligned}\right\} \rightarrow (2)$$

The parametric equation of the curve is

$$x = t,\ y = 1 - t,\ z = t$$

replacing these in (1) we have—

$$\left.\begin{aligned} \frac{t}{t-1} &= c_1 \text{ and} \\ \frac{t-1}{t} &= c_2 \end{aligned}\right\} \rightarrow (3)$$

∴ From (3) we have –

$$C_1 = \frac{1}{C_2} \Rightarrow (4)$$

Now replacing C_1 and C_2 in (4) using (2) we have —

$$\frac{x}{y} = \frac{1}{y/z}$$

$$\Rightarrow x = z$$

$$\Rightarrow x - z = 0$$

Which is the required integral surface.