EQUILIBRIUM POINTS AND PHASE PLANE

PRESENTED BY : DR. RAJIB BISWAKARMA

Assistant Professor
DEPTT. OF MATHEMATICS
Silapathar College

Def":- Phase Plane:- A pair of first order ordinary differential equations can be reduced to a single first order O.D.E.
The solution graph of this reduced O.D.E is called phase plane.

Note:- We shall consider a system of the form

$$\frac{dx}{dt} = P(x,y)$$

$$\frac{dy}{dt} = Q(x,y)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Q(x,y)}{P(x,y)} = R(x,y) \text{ (say)}$$

$$\therefore \quad \frac{dy}{dx} = R(x,y) \quad \text{---} (1)$$

If we solve eq" (1) then we get a phase plane.

Example:- Find the phase plane curve of the system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x.$$

Sol^{n}:

We know

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-x}{y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

$$\Rightarrow y\,dy = -x\,dx$$

$$\Rightarrow y\,dy + x\,dx = 0$$

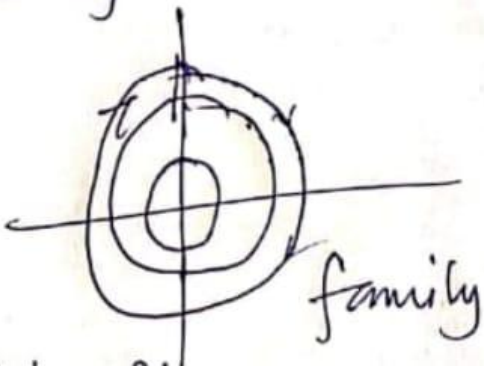
Integrating

$$\int y\,dy + \int x\,dx = \underline{c}$$

$$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} = \frac{c}{2}$$

$$\Rightarrow y^2 + x^2 = c = (k^2) \text{ say}$$

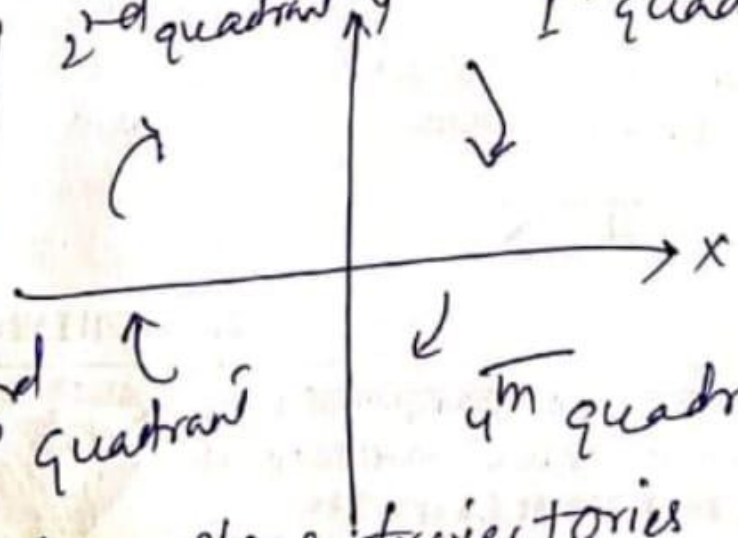
$$\Rightarrow x^2 + y^2 = k^2$$



family of circle

Trajectories and the phase-plane diagram:—

Let us consider xy-plane, we obtain a direction vector for any solution in quadrant given by the arrow.



$\frac{dx}{dt} = y > 0$,
so x is increasing

$\frac{dy}{dt} = -x < 0$
so y is decreasing.

The phase plane trajectories, move in a clockwise direction.

Equilibrium point or Critical or Singular point :-

Let. $\frac{dx}{dt} = p(x,y)$,

$\frac{dy}{dt} = q(x,y)$, be a pair of 1st order diff. eqn.

a point (x_0, y_0) at which both $p(x_0,y_0) = 0$, and $q(x_0,y_0) = 0$ is called a equilibrium point.

THANK YOU