Vector Potential:

The concept of magnetic vector potential is very useful in studying radiation from Antennas, waveguides and microwaves.

we know, divergence of magnetic flux density is zero. i.e.

$$\vec{\nabla}\cdot\vec{B} = 0 \longrightarrow (1)$$

we know, the divergence of curl of any vector field is zero. i.e.

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0 \longrightarrow (2)$$

Comparing (1) and (2) :-

$$\vec{B} = \vec{\nabla}\times\vec{A} \rightarrow (3)$$

where, $\vec{A}$ is called magnetic vector potential.

Thus, the curl of magnetic vector potential gives magnetic flux density.

Note: S.I. unit of $\vec{A}$ is wb/m2.

Expression for magnetic vector potential:

From Biot-Savart's law:—

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \vec{r}}{r^3} \longrightarrow (1)$$

The current element is expressed as:—

$$I d\vec{l} = \frac{I}{A}.A.d\vec{l} = \vec{J}.dv \longrightarrow (2)$$

where, dv is the volume element

$A \to$ Area of cross-section.

$$(1) \Rightarrow \quad d\vec{B} = \frac{\mu_0}{4\pi}\frac{(\vec{J} \times \vec{r})}{r^3} dv \longrightarrow (3).$$

We know,

$$\vec{\nabla}.\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3} \longrightarrow (4)$$

$$(3) \Rightarrow \quad d\vec{B} = \frac{\mu_0}{4\pi} \times \vec{J} \times \left(\frac{\vec{r}}{r^3}\right) dv$$

$$= \frac{\mu_0}{4\pi} \vec{J} \times \left(-\vec{\nabla}\left(\frac{1}{r}\right)\right) dv \longrightarrow (5)$$

$$= -\frac{\mu_0}{4\pi} \vec{J} \times \vec{\nabla}\left(\frac{1}{r}\right) dv \longrightarrow (5)$$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\left\{\vec{\nabla}\left(\frac{1}{r}\right) \times \vec{j}\right\} . dv \rightarrow (5)$$

Since $(\vec{A} \times \vec{B} = -\vec{B} \times \vec{A})$

Now,

$$\vec{\nabla} \times (\phi \vec{c}) = \phi(\vec{\nabla} \times \vec{c}) + (\vec{\nabla}\phi) \times \vec{c} \quad \text{(formula)}$$

Here, $\phi = \frac{1}{r}$, $\vec{c} = \vec{j}$

$$\Rightarrow \vec{\nabla} \times \left(\frac{\vec{j}}{r}\right) = \frac{1}{r}(\vec{\nabla} \times \vec{j}) + \left\{\vec{\nabla}\left(\frac{1}{r}\right) \times \vec{j}\right\}$$

$$\Rightarrow \vec{\nabla}\left(\frac{1}{r}\right) \times \vec{j} = \vec{\nabla} \times \left(\frac{\vec{j}}{r}\right) - \frac{1}{r}(\vec{\nabla} \times \vec{j}) \rightarrow (6a)$$

Since, $\vec{\nabla} \times \vec{J} = 0$ for steady current and moving in a straight wire.

$$\Rightarrow \vec{\nabla}\left(\frac{1}{r}\right) \times \vec{j} = \vec{\nabla} \times \left(\frac{\vec{j}}{r}\right) \rightarrow (6)$$

using (6) in (5) $\Rightarrow$

$$(5) \Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\left\{\vec{\nabla} \times \left(\frac{\vec{j}}{r}\right)\right\} . dv$$

$$\Rightarrow d\vec{B} = \vec{\nabla} \times \left(\frac{\mu_0 \vec{j}}{4\pi r}\right) dv$$

$$\Rightarrow d\vec{B} = \vec{\nabla} \times \left(\frac{\mu_0 \vec{j}}{4\pi r}\right) dv \rightarrow (7)$$

The total magnetic field at any point is given by:

$$\cancel{\vec{B} = \iiint}$$

$$\vec{B} = \iiint_V \vec{\nabla} \times \left(\frac{\mu_0 \vec{j}}{4\pi r}\right) dv$$

$$\Rightarrow \vec{B} = \iiint_V \vec{\nabla} \times \left(\frac{\mu_0 \vec{j}}{4\pi r}\right) dv \longrightarrow (8)$$

$$\Rightarrow \vec{B} = \vec{\nabla} \times \iiint_V \left(\frac{\mu_0 \vec{j}}{4\pi r}\right) dv \longrightarrow (9)$$

Since $\vec{B} = \vec{\nabla} \times \vec{A} \rightarrow (10)$

From (9) & (10) $\Rightarrow$

$$\vec{A} = \iiint_V \frac{\mu_0 \vec{j}}{4\pi r}\, dv$$

$$\Rightarrow \vec{A} = \frac{\mu_0}{4\pi} \iiint_V \frac{\vec{j}}{r} \, dv \quad \rightarrow (11)$$

It gives the magnetic vector potential due to volume current distribution.

Current loop as a Magnetic dipole and its dipole moment (Analogy with Electric dipole) :—