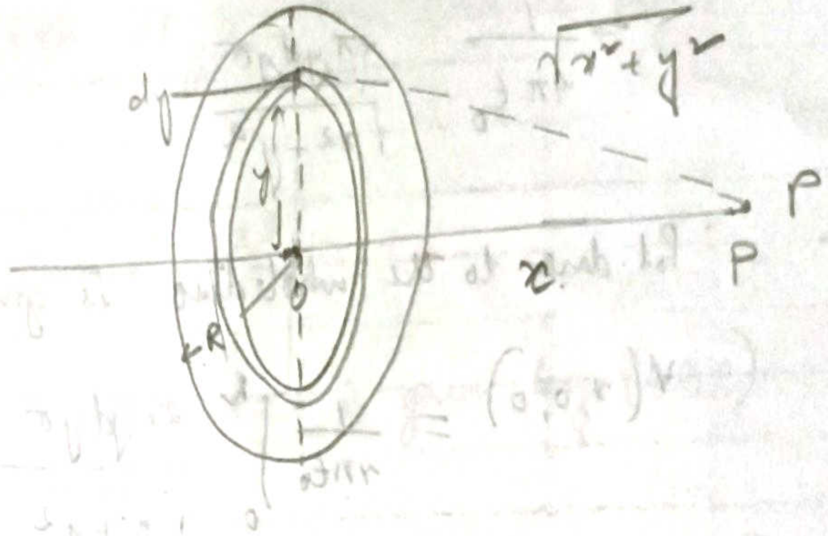
Q1. a) Find the electric potential at a point on the axis of a uniformly charged disc of surface charge density σ and radius R. 8

Ans :



Let us consider a disc lying in the Y-Z plane. 'P' be a pt. lying on the X-axis at a distance x from the centre. X-axis has been taken as the axis of symmetry of the disc. The disc may be supposed to be formed by a large number of thin concentric ring shaped elements. Consider one such ring of radius y and thickness dy. (dA)

The area of the ring element $= 2\pi y\,dy$

The charge on this element, $dq = 2\pi y\,dy\,\sigma$; $[\sigma \times dA]$

[σ is the surface charge density] dist. of all points lying on the ring from P is $\sqrt{x^2+y^2}$

$\therefore$ Potential at P due to the ring is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2\pi y\,dy\,\sigma}{\sqrt{x^2+y^2}}$$

$\therefore$ Pot. due to the whole disc is given by

$$V(x,0,0) = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{2\pi y\,dy\,\sigma}{\sqrt{x^2+y^2}}$$

$$= \frac{\cancel{2\pi}\sigma}{\cancel{4\pi}_{2}\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$= \frac{\sigma}{2\epsilon_0}\left|(x^2+y^2)^{1/2}\right|_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[(R^2+x^2)^{1/2} - x\right] \quad x > 0.$$

This gives the potential at a point on the axis of a uniformly charged disc of uniform surface charge density σ

Show graphically how the electric potential varies with distance of the point from the centre. 2

Ans: We have, the expression for electric potential at a point on the axis at a certain distance from the centre as

$$V = \frac{\sigma}{2\epsilon_0}\left[\left(R^2 + z^2\right)^{1/2} - \cancel{R}\right]$$

At the centre of the disc, $z = 0$ is given by $(0,0,0)$

$$= \frac{\sigma}{2\epsilon_0} R$$

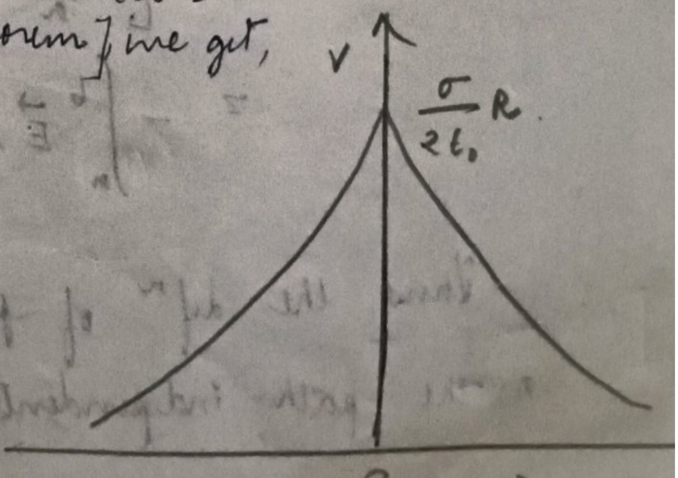
The potential goes on decreasing when we move away from the centre of the disc and at a point sufficiently far away it is given by

$$\frac{\sigma z}{2\epsilon_0}\left[\left(1 + \frac{R^2}{z^2}\right)^{1/2} - 1\right] = \frac{\sigma z}{2\epsilon_0}\left[1 + \frac{1}{2}\frac{R^2}{z^2} - 1\right]$$

[by applying Binomial theorem] we get,

$$V = \frac{\sigma z}{4\epsilon_0}\frac{R^2}{z^2} = \frac{1}{4\pi\epsilon_0}\frac{\pi\sigma R^2}{z}$$

The graph betn z and V is shown as



c) Establish that, for an electric field due to a static charge, $\boxed{\vec{E} = -\nabla V}$

Ans:

Let us define the electric potential at a location P as $V(P) = -\int_{O}^{P} \vec{E}\cdot d\vec{l}$ $\longrightarrow$ (1) where O is a certain origin

[by convention, origin is taken at infinity]

Then the pot. diff. betn two pts a & b can be replaced as

$$V(b) - V(a) = -\int_{O}^{b} \vec{E}\cdot d\vec{l} + \int_{O}^{a} \vec{E}\cdot d\vec{l}$$

$$= -\int_{O}^{b} \vec{E}\cdot d\vec{l} - \int_{a}^{O} \vec{E}\cdot d\vec{l}$$

$$= -\left[\int_{a}^{O} \vec{E}\cdot d\vec{l} + \int_{O}^{b} \vec{E}\cdot d\vec{l}\right]$$

$$= -\int_{a}^{b} \vec{E}\cdot d\vec{l} \longrightarrow (2)$$

Thus, the defn of potential difference is based on the path independents of the integration in the RHS of eqn ②. In other words, it is due to the cond

that $\boxed{\vec{\nabla} \times \vec{E} = 0}$

Now, from the ~~divergence~~ theorem of gradient, we have,

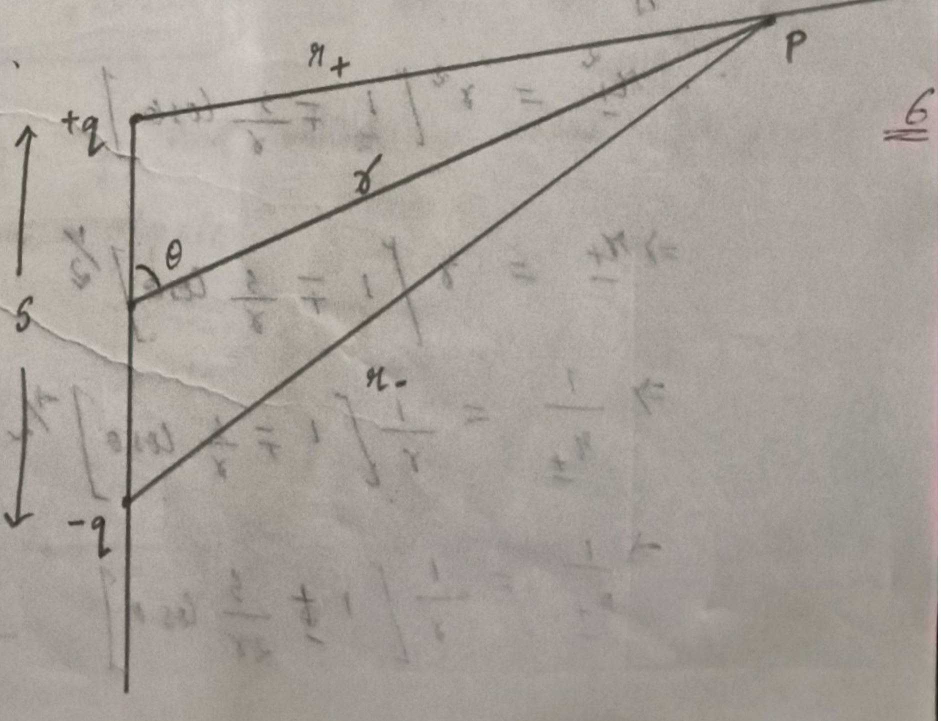
$$V(b) - V(a) = \int_a^b (\vec{\nabla} V) \cdot d\vec{l} \longrightarrow ③$$

Equating ② & ③, we get

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

2.13. Show that the potential due to an electric dipole is
$V(P) = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot \vec{r}}{r^2}$ where the symbols denote their usual meaning.

Ans:



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In the figure, P is an arbitrary point near an electric dipole. The relevant parameters have been explained in the figure.

The electric potential at P on account of the dipole is

$$V(P) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+} - \frac{q}{r_-}\right] \longrightarrow (1)$$

w.r.t the figure,

$$r_{\pm}^2 = r^2 + (s/2)^2 \mp 2r\frac{s}{2}\cos\theta$$

$$= r^2 + \frac{s^2}{4} \mp rs\cos\theta$$

$$= r^2\left[1 + \frac{s^2}{4r^2} \mp \frac{s}{r}\cos\theta\right] \longrightarrow (2)$$

In the context of a dipole $r \gg s$, so the term $\frac{s^2}{4r^2}$ can be neglected.

$$\therefore r_{\pm}^2 = r^2\left[1 \mp \frac{s}{r}\cos\theta\right]$$

$$\Rightarrow r_{\pm} = r\left[1 \mp \frac{s}{r}\cos\theta\right]^{1/2}$$

$$\Rightarrow \frac{1}{r_{\pm}} = \frac{1}{r}\left[1 \mp \frac{s}{r}\cos\theta\right]^{-1/2}$$

$$\Rightarrow \frac{1}{r_{\pm}} = \frac{1}{r}\left[1 \pm \frac{s}{2r}\cos\theta\right] \longrightarrow (3)$$

[making a binomial expansion, retaining only the first 2 terms]

$\because \frac{s}{r} << 1$.

$\therefore \frac{1}{r_+} - \frac{1}{r_-} = \frac{1}{r}\left[1 + \frac{s}{2r}\cos\theta\right] - \frac{1}{r}\left[1 - \frac{s}{2r}\cos\theta\right]$

$$\frac{1}{r_+} - \frac{1}{r_-} = \frac{s\cos\theta}{r^2} \longrightarrow (4).$$

Substituting (4) in eq^n (1), we get

$$V(P) = \frac{q}{4\pi\epsilon_0}\,\frac{s\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{p\cos\theta}{r^2} \quad [qs = p = \text{dipole moment}]$$

$$\boxed{V(P) = \frac{1}{4\pi\epsilon_0}\,\frac{\vec{p}\cdot\hat{r}}{r^2}} = \frac{1}{4\pi\epsilon_0}\,\frac{\vec{p}\cdot\vec{r}}{r^3}$$

which is the reqd exp^n and which is valid for points far away from a dipole.