Numerical Integration

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Numerical Integration

General Quadrature formula.

Let, $I = \int_a^b f(x)\,dx$ be a definite ~~interval~~ integral. we devide the interval $[a,b]$ in n equal sub-interval each of length h. i.e. $h = \frac{b-a}{n}$. Than,

Possible value of x is

$x_0 = a$, $x_0 + h = x_1$, $x_0 + 2h = x_2$, ... $x_0 + nh = b$.

corresponding values of y is

$f(x_0)$, $f(x_0 + h)$, ... $f(x_0 + nh)$.

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx.$$

Again, $u = \frac{x - x_0}{h}$

$\Rightarrow x = x_0 + hu$

$\Rightarrow dx = h\,du$.

$\therefore$ ~~$\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x_0+hu)\,h\,du$~~. $\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx$.

$$\int_a^b f(x)\,dx = \int_0^n f(x_0 + hu)\,h\,du.$$

$$= h \int_0^n f(x_0 + hu)\,du.$$

$$= h \int_0^n \left[y_0 + \Delta y_0 u + \frac{\Delta^2 y_0}{2!} u(u-1) + \frac{\Delta^3 y_0}{3!} u(u-1)(u-2) + \dots \right] du$$

$= h\left[\cancel{y_0 u + \Delta y_0} \frac{u^2}{2} + \frac{\Delta^2 y_0}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right) + \frac{\Delta^3 y_0}{6}\left(\frac{u^4}{4} - \frac{3u^3}{3} \right.\right.$

Ex

$$\int_a^b f(x)dx = h\left[y_0 u + \Delta y_0 \frac{u^2}{2} + \frac{\Delta^2 y_0}{2}\left(\frac{u^3}{3}-\frac{u^2}{2}\right) + \frac{\Delta^3 y_0}{6}\left(\frac{u^4}{4} - 3\frac{u^3}{3} + 2\frac{u^2}{2}\right) + \cdots\right]_0^n$$

$$\int_{x_0}^{x_0+nh} f(x)dx = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3}-\frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2} + \left(\frac{n^4}{4} - \frac{3n^3}{3} + 2\frac{n^2}{2}\right)\frac{\Delta^3 y_0}{6} + \cdots\right]$$

$$\Rightarrow \int_{x_0}^{x_0+nh} f(x)dx = h\left[ny_0 + \Delta y_0 \frac{n^2}{2} + \frac{\Delta^2 y_0}{2}\left(\frac{n^3}{3}-\frac{n^2}{2}\right) + \frac{\Delta^3 y_0}{6}\left(\frac{n^4}{4} - n^3 + n^2\right) + \cdots\right]$$

TRAPEZOIDAL RULE.

Let, $I = \int_a^b f(x)\,dx$ be a definite integral. We divide the interval $[a,b]$ in n equal sub intervals each of length $h = \frac{b-a}{n}$, then Possible value of x is $x_0 = a$, $x_1 = x_0 + h$, $x_2 = x_0 + 2h$ .... $x_0 + nh = b$.

corresponding value of y is –

$f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$ .... $f(x_0+nh)$.

$$\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx.$$

Again,

$$u = \frac{x - x_0}{h}$$

$$\Rightarrow x = uh + x_0.$$

$$\Rightarrow dx = h\,du.$$

$$\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx.$$

$$= \int$$

$$= \int_0^n f(x_0+hu)\, h\,du.$$

$$= h\int_0^n f(x_0+hu)du.$$

$$= h\int_0^n \left[y_0 + \Delta y_0 u + \frac{\Delta^2 y_0}{2!} u(u-1) + \frac{\Delta^3 y_0}{3!} u(u-1)(u-2) + \cdots \right] du$$

$$= h\left[y_0 u + \Delta y_0 \frac{u^2}{2} + \frac{\Delta^2 y_0}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right) + \frac{\Delta^3 y_0}{6}\left(\frac{u^4}{4} - 3\frac{u^3}{3} + 2\frac{u^2}{2}\right) + \cdots \right]$$

$$\Rightarrow \int_{x_0}^{x_0+nh} f(x)dx = h\left[n y_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 y_0}{6} + \cdots \right]$$

Now,

Put, $n=1$ in above general quadrature formula and neglect the difference above first order.

$$\int_{x_0}^{x_0+h} f(x)dx = h\left[y_0 + \frac{1^2}{2}\Delta y_0 \right]$$

$$= h\left[y_0 + \frac{1}{2}(y_1 - y_0) \right]$$

$$\Rightarrow \int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2}\left[y_0 + y_1 \right]. \quad \text{—— (1)}$$

For the next sub. interval $[x_1, x_2]$

$$\int_{x_0+h}^{x_0+2h} f(x)dx = h\left[y_1 + \frac{1}{2}\Delta y_1 \right]$$

$$= h\left[y_1 + \frac{1}{2}(y_2 - y_1) \right] = \frac{h}{2}\left[y_1 + y_2 \right] \quad \text{—— (2)}$$

Similarly,

$$\int_{x_0+2h}^{x_0+3h} f(x)\,dx = \frac{h}{2}[y_2 + y_3] \quad \text{--- (3)}$$

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$$\int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = \frac{h}{2}[y_{n-1} + y_n] \quad \text{--- (n)}$$

Adding (1) to (n).

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

This is the Trapezoidal Rule.

Ex-11 Find the integration by Trapezoidal rule using 6 equal parts.

$$\int_0^1 \frac{1}{1+x}\,dx$$

Soln:-

$x_0 = 0$, $x_0 + nh = 1$

$\Rightarrow nh = 1$.

If $n = 6$.

$\therefore h = 1/6$

$f(x) = \frac{1}{1+x}$

x	$x_0 = 0$	$x_1 = \frac{1}{6}$	$x_2 = 2/6$	$x_3 = \frac{3}{6}$	$x_4 = \frac{4}{6}$	$x_5 = 5/6$	$x_6 = 1$
$y = \frac{1}{1+x}$	$y_0 = 1$	$y_1 = \frac{6}{7}$	$y_2 = \frac{6}{8}$	$y_3 = \frac{6}{9}$	$y_4 = \frac{6}{10}$	$y_5 = \frac{6}{11}$	$y_6 = \frac{6}{12}$

$$\therefore \int_0^1 \frac{dx}{1+x} = \frac{1}{12}\left[1+\frac{1}{2}+2\left[\frac{6}{7}+\frac{6}{8}+\frac{6}{9}+\frac{6}{10}+\frac{6}{11}\right]\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2\times(0.8571 + 0.75 + 0.666 + 0.6 + 0.5454)\right]$$

$$= \frac{1}{12}\left[1.5 + 6.837\right]$$

$$= 0.694.$$

Ex-32 Find the integration $\int_2^3 \frac{dx}{x}$ by Trapezoidal rule using 6 equal part.

Solⁿ:- $x_0 = 2.$ $x_0 + nh = 3,$ $\Rightarrow nh = 1$

If $n = 6$

$h = \frac{1}{6}$.

x	$x_0 = 2$	$x_1 = \frac{13}{6}$	$x_2 = \frac{14}{6}$	$x_3 = \frac{15}{6}$	$x_4 = \frac{16}{6}$	$x_5 = \frac{17}{6}$	$x_6 = 3$
$y = \frac{1}{x}$	$y_0 = \frac{1}{2}$	$y_1 = \frac{6}{13}$	$y_2 = \frac{6}{14}$	$y_3 = \frac{6}{15}$	$y_4 = \frac{6}{16}$	$y_5 = \frac{6}{17}$	$y_6 = \frac{1}{3}$

we know,

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\right].$$

$$\Rightarrow \int_2^3 \frac{dx}{x} = \frac{1}{12}\left[\frac{1}{2}+\frac{1}{3} + 2\left[\frac{6}{13}+\frac{6}{14}+\frac{6}{15}+\frac{6}{16}+\frac{6}{17}\right]\right]$$

$$= \frac{1}{12}\left[\frac{5}{6} + 2\times(0.4615 + 0.4285 + 0.4 + 0.375 + 0.352)\right]$$

$$= \frac{1}{12}\left[\frac{5}{6} + 2\times 2.0179\right]$$

$$= \frac{1}{12}\left[\frac{5}{6} + 4.035\right] = \frac{4.868}{12} = 0.4056$$

Ex-13 $\displaystyle\int_0^6 \frac{dx}{1+x^2}$, find by Trapezoidal Rule.

Soln

$x_0 = 0$, $x_0 + nh = 6$. $\Rightarrow nh = 6$.

Lt, $n = 6$

$\therefore h = 1$

x	$x_0 = 0$	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$	$x_4 = 4$	$x_5 = 5$	$x_6 = 6$
$y = \frac{1}{1+x^2}$	$y_0 = 1$	$y_1 = \frac{1}{2}$	$y_2 = \frac{1}{5}$	$y_3 = \frac{1}{10}$	$y_4 = \frac{1}{17}$	$y_5 = \frac{1}{26}$	$y_6 = \frac{1}{37}$

$\because \displaystyle\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\right]$

$= \frac{1}{2}\left[1 + \frac{1}{37} + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26}\right)\right]$

$= \frac{1}{2}\left[\frac{38}{37} + 2(0.5 + 0.2 + 0.1 + 0.058 + .038)\right]$

$= \frac{1}{2}\left[1.027 + 2 \times .896\right]$

$= \frac{2.819}{2}$

$= 1.41$

Simpson $\frac{1}{3}$rd Rule.

Let,

$$I = \int_a^b f(x)\,dx$$ be a definite integral.

we divide the interval $[a,b]$ in n equal sub-interval each of length $h = \frac{b-a}{n}$. Then Possible value of x is

$x_0 = a$, $x_0 + h = x_1$, $x_0 + 2h = x_2$ .... $x_0 + nh = b$.

corresponding value of y is –

$f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$ ... $f(x_0+nh)$.

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx.$$

Again,

$$U = \frac{x - x_0}{h}$$

$$\Rightarrow x = x_0 + hU$$

$$\Rightarrow dx = h\,du.$$

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx.$$

$$= \int_0^n f(x_0 + hU)\,h\,du.$$

$$= h.\left[\int_0^n f(x_0+hU)\,du\right]$$

$$= h\int_0^n \left[y_0 + \Delta y_0 U + \frac{\Delta^2 y_0}{2!}U(U-1) + \frac{\Delta^3 y_0}{3!}U(U-1)(U-2) + \cdots\right] du$$

$$= h\left[y_0 U + \Delta y_0 \frac{U^2}{2} + \frac{\Delta^2 y_0}{2!}\left(\frac{U^3}{3} - \frac{U^2}{2}\right) + \frac{\Delta^3 y_0}{3!}\left(\frac{U^4}{4} - \frac{3U^3}{3} + 2\frac{U^2}{2}\right)\right]_0^n$$

$$\int_{x_0}^{x_0+nh} f(x)\,dx = h\left[n y_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 y_0}{3!} + \cdots\right]$$

Put $n=2$ and neglect third and higher differences.

$$\int_{x_0}^{x_0+2h} y\,dx = h\left[2y_0 + \frac{2^2}{2}\Delta y_0 + \left(\frac{2^3}{3} - \frac{2^2}{2}\right)\frac{\Delta^2 y_0}{2!}\right]$$

$$= h\left[2y_0 + 2(y_1 - y_0) + (y_2 - 2y_1 + y_0)\frac{2}{3}\cdot\frac{1}{2}\right]$$

$$= h\left[2y_0 + 2y_1 - 2y_0 + \frac{1}{3}(y_2 - 2y_1 + y_0)\right]$$

$$= \frac{h}{3}\left[6y_1 + y_2 - 2y_1 + y_0\right]$$

$$= \frac{h}{3}\left[y_0 + 4y_1 + y_2\right] \quad \text{— (1)}$$

Similarly,

$$\int_{x_0+2h}^{x_0+4h} y\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4\right] \quad \text{— (2)}$$

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$$\int_{x_0+(n-2)h}^{x_0+nh} y\,dx = \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n\right] \quad \text{— (n)}$$

Adding (1) to (n)

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{3}\left[y_0 + y_n + 2(y_2 + y_4 + \cdots) + 4(y_1 + y_3 + \cdots)\right]$$

This is Simpson's $\frac{1}{3}$rd Rule.

Ex-14 Find the integration $\int_2^{10} \frac{dx}{1+x}$ by taking 8 equal parts by Simpson's $\frac{1}{3}$ rd rule.

Soln:- $x_0 = 2$, $x_0 + nh = 10 \Rightarrow nh = 8$

Given, $n = 8$.

$\therefore h = 1$.

x	$x_0 = 2$	$x_1 = 3$	$x_2 = 4$	$x_3 = 5$	$x_4 = 6$	$x_5 = 7$	$x_6 = 8$	$x_7 = 9$	$x_8 = 10$
$y = \frac{1}{1+x}$	$y_0 = \frac{1}{3}$	$y_1 = \frac{1}{4}$	$y_2 = \frac{1}{5}$	$y_3 = \frac{1}{6}$	$y_4 = \frac{1}{7}$	$y_5 = \frac{1}{8}$	$y_6 = \frac{1}{9}$	$y_7 = \frac{1}{10}$	$y_8 = \frac{1}{11}$

we know,

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + 2(y_2 + y_4 + \dots) + 4(y_1 + y_3 + \dots)\right]$$

$$= \frac{1}{3}\left[\frac{1}{3} + \frac{1}{11} + 2\left(\frac{1}{5} + \frac{1}{7} + \frac{1}{9}\right) + 4\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right)\right]$$

$$= \frac{1}{3}\left[\frac{14}{33} + 2(0.2 + 0.14 + 0.11) + 4(0.25 + 0.16 + 0.125 + .1)\right]$$

$$= \frac{1}{3}(0.4242 + 2 \times 0.45 + 4 \times 0.635)$$

$$= \frac{1}{3}(0.442 + 0.9 + 2.54)$$

$$= \frac{3.882}{3}$$

$$= 1.294.$$

Ex-15 $\int_0^1 e^x\,dx$ by Simpson's $\frac{1}{3}$rd rule taking $h = .1$

x	$x_0 = 0$	$x_1 = .1$	$x_2 = .2$	$x_3 = .3$	$x_4 = .4$	$x_5 = .5$	$x_6 = .6$	$x_7 = .7$	$x_8 = .8$	$x_9 = .9$	$x_{10} = 1$
$= e^x$	$y_0 = 1$ ~~$y_0 = 1.105$~~	$y_1 = 1.105$ ~~$y_1 = 1.221$~~	$y_2 = 1.221$ ~~$y_2 = 1.3498$~~	$y_3 = 1.3498$	$y_4 = 1.4918$	$y_5 = 1.6487$	$y_6 = 1.8221$	$y_7 = 2.0137$	$y_8 = 2.2255$	$y_9 = 2.4596$	$y_{10} = 2.718$

We know

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{3}\left[y_0 + y_n + 2(y_2+y_4+\cdots) + 4(y_1+y_3+\cdots)\right]$$

$$= \frac{.1}{3}\left[1 + 2.718 + 2(1.221 + 1.4918 + 1.8221 + 2.2255) + 4(1.105 + 1.3498 + 1.6487 + 2.0137 + 2.4596)\right]$$

$$= \frac{.1}{3}\left[3.718 + 2 \times 6.7604 + 4 \times 8.5768\right]$$

$$= \frac{.1}{3}\left[3.718 + 13.5208 + 34.3072\right]$$

$$= 1.7182.$$

Ex-16 $\int_4^{5.2} \log_e x\,dx$ by Simpson $\frac{1}{3}$rd rule taking 6 equal int.

Soln:- Here, $x_0 = 4$ $\quad x_0 + nh = 5.2 \Rightarrow nh = 1.2$

Given, $n = 6.$

$h = .2$

x	$x_0 = 4$	$x_1 = 4.2$	$x_2 = 4.4$	$x_3 = 4.6$	$x_4 = 4.8$	$x_5 = 5$	$x_6 = 5.2$
$y = \log_e x$	$y_0 = 1.3862$	$y_1 = 1.4350$	$y_2 = 1.4816$	$y_3 = 1.5260$	$y_4 = 1.5686$	$y_5 = 1.6094$	$y_6 = 1.6486$

$$\therefore \int_{4}^{5.2} \log_e x\,dx = \frac{\cdot 2}{3}\left[1.3862 + 1.6486 + 2(1.4816 + 1.5686) + 4(1.\right.$$

$$= \frac{\cdot 2}{3}\left[1.3862 + 1.6486 + 6.1004 + 18.2816\right.$$

$$= 0.0666 \times 27.4168$$

$$= \cancel{1.8257} = 1.828$$

Simpson $\frac{3}{8}$ Rule:

Let, $I = \int_a^b f(x)\,dx$ be a definite integral.

we divide the interval $[a,b]$ in n equal sub-interval each of length, $h = \frac{b-a}{n}$. Then Possible value of x is

$x_0 = a$, $x_0 + h = x_1$, $x_0 + 2h = x_2$ . . . . . $x_0 + nh = b$.

corresponding value of y is –

$f(x_0)$, $f(x_0 + h)$, $f(x_0 + 2h)$ . . . . $f(x_0 + nh)$.

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx.$$

Again, $U = \frac{x - x_0}{h}$

$$\Rightarrow x = x_0 + hU.$$

$$\Rightarrow dx = h\,dU.$$

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx.$$

$$= \int_0^n f(x_0 + hU)\,h\,du.$$

$$= h \int_0^n f(x_0 + hU)\,du.$$

$$\int_a^b f(x)\,dx = h\int_0^n \left[y_0 + \Delta y_0 u + \frac{\Delta^2 y_0}{2!} u(u-1) + \frac{\Delta^3 y_0}{3!} u(u-1)(u-2) + \dots \right] du.$$

$$= h\left[y_0 u + \Delta y_0 \frac{u^2}{2} + \frac{\Delta^2 y_0}{2!}\left(\frac{u^3}{3} - \frac{u^2}{2}\right) + \frac{\Delta^3 y_0}{3!}\left(\frac{u^4}{4} - 3\frac{u^3}{3} + 2\frac{u^2}{2}\right) + \dots \right]_0^m$$

$$\int_{x_0}^{x_0+nh} f(x)\,dx = h\left[n y_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 y_0}{3!} + \dots \right]$$

Put $n=3$ and neglect differenence above 3.

$$\int_{x_0}^{x_0+3h} y\,dx = h\left[3y_0 + \frac{9}{2}\Delta y_0 + \left(\frac{27}{3} - \frac{9}{2}\right)\frac{\Delta^2 y_0}{2} + \left(\frac{81}{4} - 27 + 9\right)\frac{\Delta^3 y_0}{6} \right]$$

$$= h\left[3y_0 + \frac{9y_1 - 9y_0}{2} + \frac{54-27}{6}\cdot\frac{1}{2}(y_2 - 2y_1 + y_0) + \frac{81-108+36}{4}\cdot\frac{1}{6}(y_3 - 3y_2 + 3y_1 - y_0) \right]$$

$$= h\left[3y_0 + \frac{9y_1 - 9y_0}{2} + \frac{9(y_2 - 2y_1 + y_0)}{4} + \frac{3(y_3 - 3y_2 + 3y_1 - y_0)}{8} \right]$$

$$= h\left[\frac{24y_0 + 36y_1 - 36y_0 + 18y_2 - 36y_1 + 18y_0 + 3y_3 - 9y_2 + 9y_1 - 3y_0}{8} \right]$$

$$= \frac{h}{8}\left[3y_0 + 9y_1 + 9y_2 + 3y_3 \right]$$

$$= \frac{3h}{8}\left[y_0 + 3(y_1 + y_2) + y_3 \right] \quad \text{—— (1)}$$

Similarly,

$$\int_{x_0+3h}^{x_0+6h} y\,dx = \frac{3h}{8}\left[y_3 + 3(y_4+y_5) + y_6\right] \quad \text{— (ii)}$$

$$\int_{x_0+6h}^{x_0+9h} y\,dx = \frac{3h}{8}\left[y_6 + 3(y_7+y_8) + y_9\right] \quad \text{— (iii)}$$

$$\int_{x_0+(n-3)h}^{x_0+nh} y\,dx = \frac{3h}{8}\left[y_{n-3} + 3(y_{n-2}+y_{n-1}) + y_n\right] \quad \text{— (n)}$$

Adding (1) to (n).

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + y_7 + \dots) + 2(y_3 + y_6 + y_9 + \dots\right.$$

This is Simpson's $\frac{3}{8}$th rule.

Ex-17 $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{3}{8}$ Rule.

Soln $x_0 = 0$. $x_0 + nh = 1 \Rightarrow nh = 1$

Let,

$n = 6$

$h = 1/6$.

x	$x_0 = 0$	$x_1 = \frac{1}{6}$	$x_2 = \frac{2}{6}$	$x_3 = \frac{3}{6}$	$x_4 = \frac{4}{6}$	$x_5 = \frac{5}{6}$	$x_6 = 1$
$y = \frac{1}{1+x}$	$y_0 = 1$	$y_1 = \frac{6}{7}$	$y_2 = \frac{6}{8}$	$y_3 = \frac{6}{9}$	$y_4 = \frac{6}{10}$	$y_5 = \frac{6}{11}$	$y_6 = \frac{6}{12}$

$$\therefore \int_0^1 \frac{dx}{1+x} = \frac{3}{8\times 6}\left[1 + \frac{1}{2} + 3\left(\frac{6}{7} + \frac{6}{8} + \frac{6}{10} + \frac{6}{11}\right) + 2\left(\frac{6}{9}\right)\right]$$

$$= \frac{3}{48}\left[1 + .5 + 3(.857 + .75 + .6 + .5454) + 1.33\right]$$

33.2616 = 0.69

Ex-18 $\int_4^{5\cdot 2} \log x\, dx$ by Simpson's $\frac{3}{8}$th rule taking 6 equal intervals.

Soln $x_0 = 4$, $x_0 + nh = 5\cdot 2$ $\Rightarrow nh = 1\cdot 2$

Given, $n = 6$

$h = \cdot 2$

x	$x_0 = 4$	$x_1 = 4\cdot 2$	$x_2 = 4\cdot 4$	$x_3 = 4\cdot 6$	$x_4 = 4\cdot 8$	$x_5 = 5$	$x_6 = 5\cdot 2$
$y = \log x$	$y_0 = 1\cdot 386$	$y_1 = 1\cdot 435$	$y_2 = 1\cdot 481$	$y_3 = 1\cdot 526$	$y_4 = 1\cdot 568$	$y_5 = 1\cdot 609$	$y_6 = 1\cdot 648$

$$\int_4^{5\cdot 2} \log x\, dx = \frac{3 \times \cdot 2}{8} \left[(1\cdot 386 + 1\cdot 648) + 3(1\cdot 435 + 1\cdot 481 + 1\cdot 568 + 1\cdot 609) + 2 \times 1\cdot 526 \right]$$

$$= 0\cdot 075 \left[3\cdot 034 + 18\cdot 279 + 3\cdot 052 \right]$$

$$= 1\cdot 827$$

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