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# DERIVATIVES OF FUNCTION

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# Content of the Course

1. Definition of Differentiation, Differentiation at a Particular Point.
2. Standard Derivatives and Examples.
3. Differentiation of a Function of Function (Chain Rule) and Example.

# Definition of Differentiation, Differentiation at a Particular Point



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## Definition

Let $X$ and $Y$ be two non-empty sub-sets of the set of real numbers $\mathbb{R}$. Then a real valued function $f : X \to Y$ such that $y = f(x), x \in X, y \in Y$ is called differentiable at $x$ if $\frac{dy}{dx}$ or

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

provided the limit exists.

Usually $\Delta x$ is replaced by $h$ so that

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h},$$

or

$$f'(x) = \lim_{h \to 0-} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0+} \frac{f(x + h) - f(x)}{h}$$

# Definition of Differentiation, Differentiation at a Particular Point



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Here we discuss the process of finding the expression of derivative of given real valued function $y = f(x)$. Let $\Delta x$ (positive or negative) is called the increment in $x$ and $\Delta y$ (positive or negative or zero) is called the increment in $y$. So,

$$(y + \Delta y) - y = f(x + \Delta x) - f(x)$$
$$\Rightarrow \Delta y = f(x + \Delta x) - f(x)$$
$$\Rightarrow \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

This represents the ratio of increments in $y$ and $x$.

# Definition of Differentiation, Differentiation at a Particular Point



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Therefore the the limiting value of the ratio as $\Delta x \to 0$ is defined to be the derivative of $y = f(x)$ with respect to $x$ and is denoted by $\frac{dy}{dx}$ or $f'(x)$.

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Usually $\Delta x$ is replaced by $h$ so that

$$\frac{dy}{dx} \;\; or \;\; f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$

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# Differentiation at a Particular Point

f the derivative is desired at a particular point $x = a$ within the domain of $f$,
we write

$$[\frac{dy}{dx}]_{x=a} \;\; or \;\; f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

which can be expressed as

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

# Standard Derivative and Example



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## Example

Let $y = f(x) = x^n$ where $n \in \mathbb{Q}$,
then $f(x+h) = (x+h)^n$
Therefore, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$\Rightarrow f'(x) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$
$\Rightarrow f'(x) = \lim_{z\to x} \frac{z^n - x^n}{z-x}$, on putting
$x+h = z, h \to 0 \Rightarrow z \to x$
$\Rightarrow$f'(x) = n x$^{(n-1)}$, since [$\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{(n-1)}$
Thus $\frac{d}{dx}(x^n) = nx^{(n-1)}$
In particular
$\frac{d}{dx}(x) = 1, \frac{d}{dx}(x^2) = 2x^{(2-1)} = 2x, \frac{d}{dx}(x^3) = 3x^{(3-1)} = 3x^2,$
$\frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}, \frac{d}{dx}(\frac{1}{x^2}) = \frac{-2}{x^3}, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2x}$

# Standard Derivative and Example



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## Example

Let $y = f(x) = \sin x$,
then $f(x+h) = \sin(x+h)$
Therefore, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\Rightarrow$f'(x) $= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$,
$\Rightarrow$f'(x) $= \lim_{h \to 0} \frac{2\cos(\frac{x+h+x}{2})\sin(\frac{x+h-x}{2})}{h}$,
$\Rightarrow$f'(x) $= \lim_{h \to 0} \frac{\cos(x+\frac{h}{2})\sin(\frac{h}{2})}{\frac{h}{2}}$
$\Rightarrow$f'(x) $= \lim_{h \to 0} \cos(x + \frac{h}{2}) \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ on putting $\frac{h}{2} = \theta, h \to 0 \Rightarrow \theta \to 0$.
Thus $\frac{d}{dx}(\sin x) = \cos x$, since we know $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

# Differentiation of a Function of Function (Chain Rule)



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- If $y = f(u)$, $u = g(x)$ then $y = f[g(x)]$, and so $y$ is called a function of a function.
- Let $f$ and $g$ be derivable functions of $u$ and $x$ respectively. Then, $\frac{dy}{du} = f'(u)$ and $\frac{du}{dx}(x) = g'(x)$ exist.
- Let $\Delta x$ and $\Delta u$ are the corresponding increment in $x$ and $u$ as determined from $u = g(x)$.
- Again, corresponding to the increment $\Delta u$ in u, let $\Delta y$ be the increment in $y$ as determined from $y = f(u)$.

# Differentiation of a Function of Function (Chain Rule) and Examples



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Now, we can write

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u}.\frac{\Delta u}{\Delta x}$$

Let $\Delta x \to 0$ so that $\Delta u \to 0$

Then

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (\frac{\Delta y}{\Delta u}.\frac{\Delta u}{\Delta x}) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u}. \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$$

Hence

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

.

This rule is also known as *Chain rule*.

# Differentiation of a Function of Function (Chain Rule) and Examples



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The rule may be generalised for a finite number of functions of a function. Thus if $y = f(u), u = g(v), v = h(x)$ be three differentiable function so that $y$ is a function of $x$ then

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$$

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# Differentiation of a Function of Function (Chain Rule) and Examples

## Example

Find the derivative of $\sin^3 x$

**Solution:**

Let

$$y = \sin^3 x$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\sin x)^3$$

$$\Rightarrow \frac{dy}{dx} = 3(\sin x)^{3-1}\frac{d}{dx}(\sin x)$$

$$\Rightarrow \frac{dy}{dx} = 3\sin^2 x \cos x.$$

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*THANK YOU*