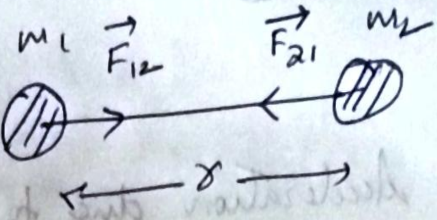
Gravitation

Important :

• Newton's law of gravitation

$F \propto m_1 m_2$

$\propto \frac{1}{r^2}$



$$\Rightarrow F = G\frac{m_1 m_2}{r^2}$$

$G \rightarrow$ constant of proportionality called gravitational constant.

If $m_1 = 1 = m_2$, $r = 1 \Rightarrow F = G$.

Thus, $G \longrightarrow$ Force of attraction between two unit masses kept unit distance apart.

Unit $\longrightarrow Nm^2/kg^2$ Value $\rightarrow 6.67\times10^{-11}\ Nm^2/kg^2$

Dimension of G :

$$G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = \frac{[M^1L^1T^{-2}][L^2]}{[M^1M^1]} = [M^{-1}L^3T^{-2}]$$

Difference

Gravitation	Gravity
Force of attraction between ~~the earth~~ any two body lying in the ~~vicinity of earth~~ universe	Force of attraction between ~~any two bodies~~ earth any any body lying in the vicinity of earth

Acceleration due to gravity

Acceleration produced on a body by the force of gravity.

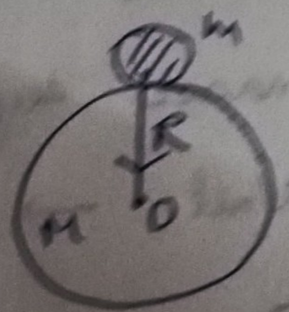
We have,

$$F = G\frac{m_1 m_2}{r^2} \quad \text{or} \quad F = \frac{GMm}{R^2}.$$

Again, $F = mg$

$$\therefore \frac{GMm}{R^2} = mg$$

$$\Rightarrow \boxed{g = G\frac{M}{R^2}}$$



$g \longrightarrow$ independent of mass of the body

Mass & density of earth

We have,

$$g = \frac{GM}{R^2}$$

$$\Rightarrow M = \frac{gR^2}{G}$$

$$= \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} \text{ kg}$$

$$= 6.018 \times 10^{24} \text{ kg}$$

Again, $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$$\Rightarrow \rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

$$= \frac{3 \times 9.8}{4 \times 3.14 \times (6.4 \times 10^6)^3}$$

$$= 5.5 \times 10^3 \text{ kg/m}^3$$

Variation of g due to shape of earth

We know, earth is not a perfect sphere.

It is flattened at poles and bulges out at equator.

Equatorial radius R_e is 21 km greater than the polar radius R_p.

$$\because \quad g = \frac{GM}{R^2}$$

$$g \propto \frac{1}{R^2}$$

$\therefore$ g $\longrightarrow$ least at equator

. Maximum at poles

Centre of gravity

- Point in the body through which the resultant of various parallel forces acting on the various particles of the body passes
- where whole weight is concentrated.

Centre of mass

- Pt. where whole mass of the body is concentrated (supposed to be)

Inertial mass

- Mass which is responsible for the inertia of a body.

$F = ma$

$\Rightarrow m = F/a$

Gravitational mass

- Mass which is responsible for the gravitational pull on the body.

$F = \frac{GMm}{R^2}$

$\Rightarrow m = \frac{FR^2}{GM} = \frac{F}{GM/R^2}$

$\Rightarrow m = F/I$

$I \rightarrow$ Intensity of gravitational field.

Gravitational field

- Space surrounding a body in which the gravitational pull of the body is realised.

Gravitational field intensity

- Force experienced by a unit mass placed at that point.

Relation between gravitational intensity and gravitational potential

$$E = \frac{GM}{r^2} \rightarrow \text{Gravitational intensity.}$$

Again $V = \frac{GM}{r} \longrightarrow$ Gravitational potential.

$$\frac{dV}{dr} = -\frac{GM}{r^2} = -E.$$

$$\Rightarrow E = -\frac{dV}{dr}.$$

Intensity is the negative gradient of potential.

Weightlessness

If the effective force due to gravity on a body is zero, then the body becomes weightless.

The weight of a body of mass 'm' is –

$$W = mg$$

If $g = 0$, then $W = 0$.

At the centre of earth, a body becomes weightless.

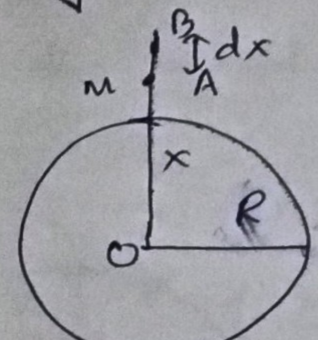
In a freely falling lift, the weight of a body becomes zero.

· Inside the satellite orbiting round the earth, weight becomes zero.

Escape velocity:-

The minimum velocity with which a body is to be projected vertically upward from the surface of the earth so that the body just crosses the gravitational field of earth and never returns to earth is called escape velocity

Derivation



Let us consider the earth to be a homogeneous sphere of mass M and radius R having centre at O.

The force of attraction on a body of mass m at A at a distance 'x' from the centre of the earth is

$$F = \frac{GMm}{x^2}$$

The work done in raising the body through a small distance dx from A to B is

$$dW = F\,dx = G\frac{Mm}{x^2}dx$$

$\therefore$ Total work done in taking the body from earth's surface $(x = R)$ to $(x = \infty)$ is

$$W = \int_R^{\infty} G \frac{Mm}{x^2} \, dx$$

$$= GMm \int_R^{\infty} x^{-2} \, dx \qquad \left[G, m, M \text{ are constants} \right]$$

$$= GMm \left[\frac{x^{-2+1}}{-2+1} \right]_R^{\infty} \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$= -GMm \left[\frac{1}{x} \right]_R^{\infty}$$

$$= -GMm \left[\frac{1}{\infty} - \frac{1}{R} \right]$$

$$W = + \frac{GMm}{R} \longrightarrow (1)$$

This work is equal to the kinetic energy supplied. If v_e is the escape velocity,

$$\frac{1}{2} m v_e^2 = \frac{GMm}{R}$$

$$\Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}} \longrightarrow (2)$$

Again,

$$g = \frac{GM}{R^2}$$

$$\Rightarrow GM = gR^2$$

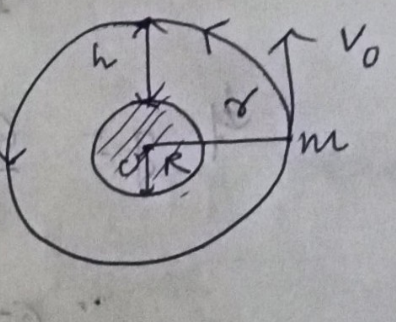
$$\therefore v_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

$$\Rightarrow \boxed{v_e = \sqrt{2gR}} \longrightarrow (3)$$

Both eqⁿ (2) & (3) are escape velocity expression.

Orbital Velocity

Minimum velocity required to put the satellite into a given orbit.



Derivation

Let a satellite of mass 'm' be revolving with an orbital velocity v_0 around the earth in circular orbit of radius r. If R is the radius of earth & 'h' is the height of the satellite above the earth's surface, then

$$r = R + h$$

The centripetal force required to keep the satellite in its orbit is

$$F_1 = \frac{mv_0^2}{r} \longrightarrow (1)$$

Again, gravitational force

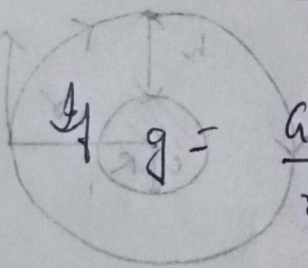
$$F_2 = \frac{GMm}{r^2} \longrightarrow (2)$$

Again,

$$F_1 = F_2$$

$$\Rightarrow \frac{mv_0^2}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}$$

$$\Rightarrow \boxed{v_0 = \sqrt{\frac{GM}{r}}} \longrightarrow (3)$$



If $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$

$$\therefore (3) \Rightarrow v_0 = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{gR^2}{R+h}}$$

$$\Rightarrow \boxed{v_0 = R\sqrt{\frac{g}{R+h}}} \longrightarrow (4)$$

eqn (3) & (4) are expression for orbital velocity.

Relation between escape & orbital velocity

$$v_o = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{GM}{R+h}}$$

when $h \approx 0$, $v_o = \sqrt{\frac{GM}{R}}$

We know $v_e = \sqrt{\frac{2GM}{R}}$

$$\therefore \boxed{\frac{v_e}{v_o} = \sqrt{2}}$$

$$\Rightarrow \boxed{v_e = \sqrt{2}\, v_o}$$

Time period of a satellite :

Time taken by the satellite to make one complete revolution around earth

$$T = \frac{2\pi r}{v_o} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{(R+h)^3}{GM}} \quad \text{or} \quad \boxed{T^2 \propto R^3}$$

Again $GM = gR^2$.

$$\Rightarrow \boxed{T = 2\pi \sqrt{\frac{(R+h)^3}{gR^2}}}$$

Height of a satellite

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\Rightarrow T^2 = 4\pi^2 \frac{(R+h)^3}{gR^2}$$

$$\Rightarrow (R+h)^3 = \frac{gR^2T^2}{4\pi^2}$$

$$\Rightarrow R+h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3}$$

$$\Rightarrow \boxed{h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3} - R}$$

Geo-stationary satellite

A satellite which always appears to be at fixed position at a definite height to an observer on earth is called a Geo-stationary satellite.

Also called Geo-synchronous satellite.

- It should be at a height of 36000 km above the earth.
- It has time period of 24 hours.

Height of a satellite

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• It moves from west to east with orbital velocity 3.1 km/sec.

Keplar's laws

1. 1st law : Law of orbit

• 2nd law : Law of area.

• 3rd law : Law of period. $T^2 \propto R^3$.