6. ELECTRIC POTENTIAL

Work done in bringing a unit positive charge from infinity to any point is termed as potential at that point i.e. if

W = work done in bringing a positive charge q_0 from infinity to that point, then , $V = \frac{W}{q_0}$

(i) Electric potential at infinity is taken to be zero.

(ii) It is not path dependent quantity if simply depends upon the starting and end points.

(iii) It is a scalar quantity.

(iv) Unit : Volt or Joule/Coulomb

(v) Dimension : $[M^1 L^2 T^{-3} A^{-1}]$

(vi) Potential due to a positive charge is positive and potential due to a negative charge is negative, hrere potential being positive and negative implies whether work is done on the charge or done by the charge respectively.

(vii) Potential due to a point charge Q at a distance r is $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

$$\Rightarrow V \alpha \frac{1}{r}$$

(viii) Total potential at a point due to a group of charges is scalar sum of individual potentials

$V_p = V_1 + V_2 + ...V_n$

(ix) Electric field is gradient of electric potential at that point. $E = -\frac{dv}{dr}$

Note : The negative sign implies that direction of electric field is in the direction of decreasing potential.

(x) Work done in bringing a charge Q from infinity to that point is

W = QV where V is potential at that point.

(xi) Potential of earth is taken to be zero.

[Allthough the earth's negatively charged sphere, yet its potential is zero. It is because of its large capacitance ($C = 4\pi\epsilon_0 R$] $C = \frac{q}{V}$

$\Rightarrow V = q/C$, $V \to 0$, as C is too large]

Example based on Electric Potential

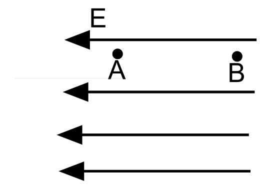
Ex.20 Can metal sphere of 1cm radius held a charge of 1 coulomb ? **[Air gets ionized at the Electric field of $E_{max} = 3 \times 10^6$ volt/m]**

Sol. No. The potential of a metal sphere of radius 1cm is given by

$$V = \frac{1}{4\pi \epsilon_0}\frac{q}{r} = 9 \times 10^9 \times \frac{1}{1\times 10^{-2}} = 9 \times 10^{11}$$

The potential is much greater then needed to ionise the air and hence the charge leakes to surrounding air. **[Air gets ionized at the potential of 3×10^6 volt]**

Ex.21 In the given diagram $V_A < V_B$ since direction of E is from B to A.



Ex.22 Infinite number of same charge q are placed at x = 1 , 2 , 4 , 8 ...... What is the potential at x = 0 ?

Sol. $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+.....\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(1-\frac{1}{2}\right)} = \frac{2q}{4\pi\varepsilon_0} = \frac{q}{2\pi\varepsilon_0}$$

$$[\therefore a + ar + ... \infty = \frac{a}{1-r_a} \quad r < 1$$

Ex.23 If the alternative charges are unlike, then what will be the potential ?

Sol. Then , $$V = \frac{1}{4\pi \epsilon_0}\left(\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+......\infty\right)$$

$$= \frac{1}{4\pi \epsilon_0}\left[\left(\frac{q}{1}+\frac{q}{2}+.....\infty\right)-\left(\frac{q}{2}+\frac{q}{8}+....\infty\right)\right]$$

$$= \frac{1}{4\pi \epsilon_0}\left[\frac{1}{1-\frac{1}{4}}-\frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right] = \frac{1}{4\pi \epsilon_0}\frac{2q}{3}$$

$$[\therefore a + ar + ar^2 ..... \infty = \frac{a}{1-r} r < 1]$$

Ex.24 A charge +q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ ... ad inf. on the x-axis, and a charge -q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ ...... ad inf. Here x_0 is a positve constant. Take the electric potential at a point due to a charge Q at a distance r from it be $Q/4\pi\epsilon_0 r$. Then , the potential at the origin due to the above system of charges is

(A) 0 (B) $\frac{q}{8\pi\epsilon_0 x_0 \log 2}$

(C) ∞ (D) $\frac{q\log 2}{4\pi\epsilon_0 x_0}$

Sol. Total potential caused by +q , at the origin

$$V_1 = \frac{kq}{x_0} + \frac{kq}{3x_0} + \frac{kq}{5x_0} + ...\infty$$

$$= \frac{kq}{x_0}\ [1 + \frac{1}{3} + \frac{1}{5} + ...\infty]$$

Total potential caused by –q , at the origin

$$V_2 = -\frac{kq}{2x_0} - \frac{kq}{4x_0} - \frac{kq}{6x_0} - .....\infty$$

$$= \frac{kq}{x_0}\left[-\frac{1}{2} - \frac{1}{4} - \frac{1}{6}.....\infty\right]$$

Net potential at the origin

$V = V_1 + V_2$

$$= \frac{kq}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - .....\infty\right]$$

$$= \frac{kq}{x_0} \log\ (1 + 1)$$

$[\because \log(1 + x) = 1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + ...\infty]$

$$= \frac{kq}{x_0}\log 2 = \frac{1}{4\pi\epsilon_0}\frac{q}{x_0}\log 2.$$

Hence answer is (D)

6.1 Potential difference :

The work done in taking a charge from one point to the other in an electric field is called the potential difference between two points.

Thus , if w be work done in moving a charge q_0 form B to A then the potential difference is given by-

$$V_A - V_B = \frac{W}{q_0}$$

(i) Unit of potential difference is volt.

(ii) This is a scalar quantity

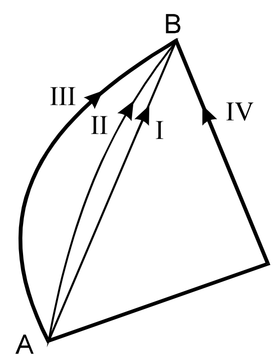
(iii) Potential difference does not depend upon Co-ordinate system

(iv) Potential difference does not depend upon the path followed. This is , because electric field is a conservative force field and work done is conservative force field does not depend upon path followed.

Ex.25 In the following fig. Along which path the work done will be maximum in carrying a charge from A to B in the presence of any another charge

Sol. Same for all the path

[Because the work done doesn't depend upon the path]



Ex.26 A charge 20μC is situated at the origin of X-Y plane. What will be potential difference between points (5a, 0) and –3a , 4a)

Sol. Distance between (0 , 0) & (5a ,a),

$r_1 = \sqrt{25a^2 + 0} = 5a$

$\therefore\ V_1 = \frac{kq}{5a}$

Distance between

(0, 0) & (–3a , 4a) $r_2 = \sqrt{9a^2 + 16a} = 5a$

$V_2 = \frac{kq}{5a}$

$\therefore V_1 - V_2 = 0$

6.2 Relationship between electric potential and intensity of electric field

(i) $V_A = -\int_{\infty}^{A} \vec{E}.\vec{dr}$, V_A = electric potential at point A .

(ii) Potential difference between two points in an electric field is given by negative value of line integral of electric field i.e.

$$V_B - V_A = -\int_{A}^{B} \vec{E}.\vec{dr}$$

(iii) $\vec{E} = -\nabla V = -$ grad

$$\nabla = \text{(gradient)} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial x}\hat{k}\right)$$

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

(iv) If v is a function of r only , then $E = -\frac{dV}{dr}$

(v) For a uniform electric field , $E = -\frac{\Delta V}{\Delta r}$ and it's direction is along the decrease in the value of V.

Ex.27 Electric potential for a point (x , y , z) is given by $V = 4x^2$ volt . Electric field at point (1 , 0 , 2) is -

Sol. $E = -\frac{dV}{dx} = -8x$

E at (1, 0 ,2)= – 8 V/m

$\therefore$ Magnitude of E

= 8V/m direction along –x axis.

Ex.28 Electric field is given by $E = \frac{100}{x^2}$ potential difference between x = 10 and x = 20 m. **[PET '89 ,94]**

Sol. $E = -\frac{dV}{dx} \Rightarrow dV = -Edx$

$$\Rightarrow \int_A^B dV = -\int_A^B E.dx$$

$$\Rightarrow V_B - V_A = -\int_{10}^{20}\frac{100}{x^2} = -5 \text{ volts}$$

Potential difference = 5 volt.

Ex.29 The potential at a point (x , 0 ,0) is given as $V = \left(\frac{1000}{x} + \frac{1500}{x^2} + \frac{500}{x^3}\right)$. What will be electric field intensity at x = 1m ?

Sol. $\therefore E = -\Delta V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

or $iE_x + jE_y + kE_z = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$$= -\frac{\partial V}{\partial x}\left[\because \frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}\right]$$

Comparing both sides

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{1000}{x} + \frac{1500}{x^2} + \frac{5000}{x^3}\right]$$

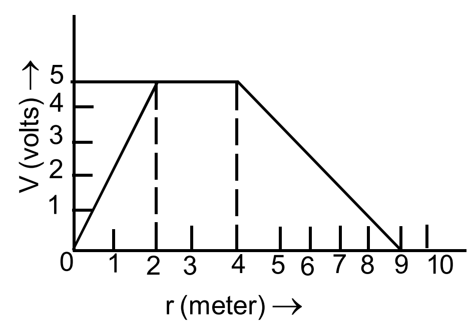
$$= -\left[-\frac{1000}{x} - \frac{2\times1500}{x^3} - \frac{3\times5000}{x^4}\right]$$

For x = 1 , (E_x) = 5500 V/m

Ex.30 In the following fig , what will be the electric field intensity at r = 3

Sol. For 2 < r < 4, V = 5 volts

$$\therefore E = -\frac{dV}{dr} = 0$$



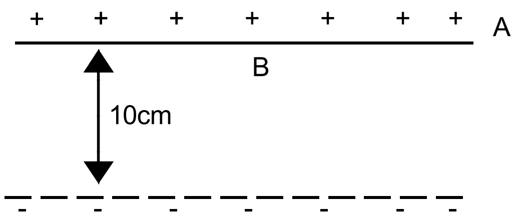
Note : In the above problem, what will value of E at r = 6 ?

at r_2 = 7m V_2 = 2 volt

at r_1 = 5m V_1 = 4 volt

$$\therefore E = -\left(\frac{V_2 - V_1}{r_2 - r_1}\right) = -\left(\frac{2-4}{7-5}\right) = 1 \text{ volt/metre}$$

Ex.31 An oil drop 'B' has charge 1.6×10^{-19}C and mass 1.6×10^{-14}kg. If the drop is in equilibrium position, then what will be the potential diff. between the plates.[The distance between the plates is 10mm]



Sol. For equilibrium, electric force = weight of drop

$\Rightarrow qE = mg$

$\Rightarrow q.\dfrac{V}{d} = mg$

$\Rightarrow V = \dfrac{mgd}{q}$

$= \dfrac{1.6\times10^{-14}\times9.8\times10\times10^{-3}}{1.6\times10^{-19}}$

$V = 10^4$ volt

[When a charged particle is in equilibrium in electric field, the following formula is often used qE = mg]

6.2 Equipotential Surface -

(i) These are the imaginary surface (drawn in an electric field) where the potential at any point on the surface has the same value.

(ii) No two equipotential surfaces ever intersects

(iii) Equipotential surfaces are perpendicular to the electric field lines

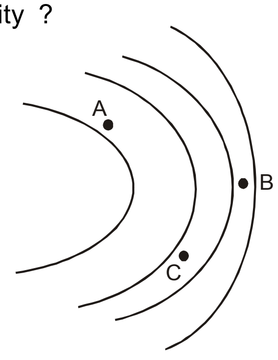
(iv) Work done in moving a charge from a one point to the other on an equipotential surface is zero irrespective of the path followed and hence there is no change in kinetic energy of the charge.

(v) Component of electric field parallel to equipotential surface is zero.

(iv) Nearer the equipotential surfaces , stronger the electric field intensity

Ex.32 Some equipotential surfaces are shown in fig.

What is the correct order of electric field intensity ?



Sol. $E_B > E_C > E_A$, because potential gradient at B is maximum.

7. POTENTIAL ENERGY OF CHARGED PARTICLE IN ELECTRIC FIELD

(i) Work done in bringing a charge from infinity to a point against the electric field is equal to the potential energy of that charge.

(ii) Potential energy of a charge of a point is equal to the product of magnitude of charge and electric potential at that point i.e. P.E. = qV

(iii) Work done in moving a charge from one point to other in an electric field is equal to change in it's potential energy i.e. work done in moving Q from A to B = $qV_B - qV_A$

$= U_B - U_A$

(iv) Work done in moving a unit charge from one point to other is equal to potential difference between two points.

Note : Circumference of the circle in above example can be considered as equipotential surface and hence work done will be zero.

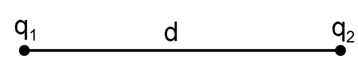
7.1 Potential Energy of System :

(i) The electric potential energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system.

(ii) If a system is given negative of it's potential energy , then all charges will move to infinity. This negative value of total energy is called the binding energy.

(iii) Energy of a system of two charges

$$PE = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{d}$$

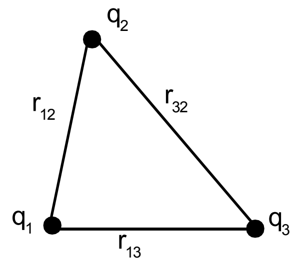


(iv) Energy of a system of three charges

$$PE = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}}+\frac{q_2q_3}{r_{23}}+\frac{q_3q_1}{r_{31}}\right]$$

(v) Energy of a system of n charges.

$$PE = \frac{1}{2}.\frac{1}{4\pi\varepsilon_0}\left(\sum_{i=1}^{n}q_i\left(\sum_{\substack{j=1\\ i\neq j}}^{n}\frac{q_j}{r_{ij}}\right)\right)$$



Note : Method to find energy of a system of n charges.

(a) Find the PE of each charge relative to all other charges.

(b) Add these all

(c) Divide the addition by 2 and resultant will be the potential energy of the system.

7.2 Work Done in An Electric Field -

(i) If electric potential at a point is V then potential energy (PE) of a charge placed at that point will be qv.

(ii) Work done in moving a charge from A to B is equal to change is PE of that charge W_{AB} = work done from A to B

$= PE_B - PE_A = q(V_B - V_A)$

(iii) Work done in moving a charge along a closed surface in an electric field is zero.

(iv) Total energy remains constant in an electric field i.e. $KE_A + PE_A = KE_B + PE_B$

KE = Kinetic energy

PE = Potential energy

(v) A free charge moves from higher PE to lower PE state in an electric field. Hence

(a) a + ve charge will move form higher potential to lower potential while,

(b) a –ve charge will move form lower potential to higher potential

(c) Work done for displacement through $\vec{r}$ for a charge experiencing a force

$$\vec{F} = W = \vec{F}.\vec{r}$$

Example based on Electric Potential Energy

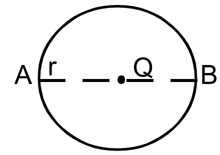
Ex.33 A charge Q is placed at the centre of a circle of a radius 'r'. Work done in taking a charge q from A to diametrically opposite point B.

Sol. Potential energy of q at A

$$= U_A = \frac{1}{4\pi\in_0}\frac{Qq}{r}$$

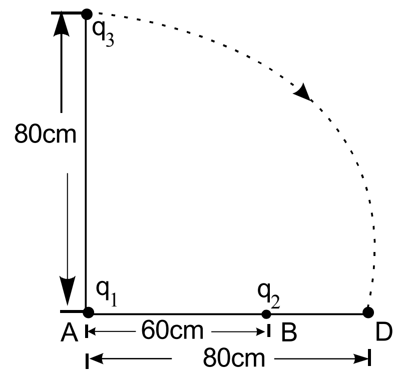
PE of q at B

$$= U_B = \frac{1}{4\pi\in_0}\frac{Qq}{r}$$



$\therefore$ Work done $= U_B - U_A = 0$

Ex.34 What will be change in potential energy of q_3, in moving it along CD for the following fig.



Sol. Potential energy of q_3 at C

[where $q_1 = 2 \times 10^{-8}C$,

$q_2 = 0.4 \times 10^{-8}$ C , $q_3 = 0.2 \times 10^{-8}$ C]

$$U_c = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{1}\right]$$

[$\therefore$ BC $= \sqrt{80^2 + 60^2}$ cm $= \sqrt{10^4}$ cm

$= 10^2$ cm = 1m]

Potential energy of q_3 at D,

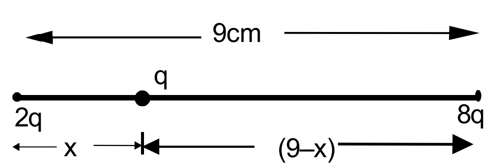
$$U_D = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{0.2}\right]$$

$$\therefore U_D - U_C = kq_2q_3\left[\frac{1}{0.2} - \frac{1}{1}\right]$$

$= 9 \times 10^9 \times 0.4 \times 10^{-8} \times 0.2 \times 10^{-8} \times 4$

$= 2.88 \times 10^{-7}$ Joule

Ex.35 In the following fig, where the charge 'q' must be kept , so that the potentail energy of the system will be minimum.?



Sol. Suppose the charge q is placed at distance x from 2q.Potential energy of the system

$$U = k\left[\frac{2qq}{X\times10^{-2}}+\frac{8qq}{(9-X)\times10^{-2}}+\frac{2q\times8q}{9\times10^{-2}}\right]$$

For U to be minimum $\frac{\partial U}{\partial X} = 0$ which gives X = 3cm.

Ex.36 The charges of 10μc each are kept at three corners of an equilateral triangle of 10cm side. What is the potential energy of the system ?

Sol. PE of 1 = $U_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

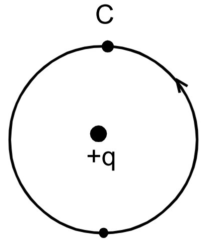
PE of 2 = $U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

PE of 3 = $U_3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

PE of system= $\frac{1}{2}(U_1 + U_2 + U_3) = \frac{3}{4\pi\varepsilon_0}.\frac{q^2}{r}$

$$= \frac{3\times9\times10^9\times(10\times10^{-6})^2}{10\times10^{-2}} = 27 \text{ Joule.}$$

Ex.37 A charge +q is placed at the centered of a circles. What will be the amount of work done in carrying a charge q' from B to C in the fig.



Sol. Zero. Because circular path is a equipotential surface

Hence $V_B - V_C = 0$

$\therefore$ W = q' $(V_B - V_D) = 0$

Ex.38 An electron (mass m , charge e) is accelerated through a potential difference of V volt. Find the final velocity of electron.

Sol. $KE_i = 0$

$PE_i = eV_1$

$KE_f = 1/2\ mv^2$

$PE_f = eV_2$ $\begin{cases} i = \text{initial} \\ f = \text{final}\end{cases}$

$KE_i + PE_i = KE_f + PE_f$

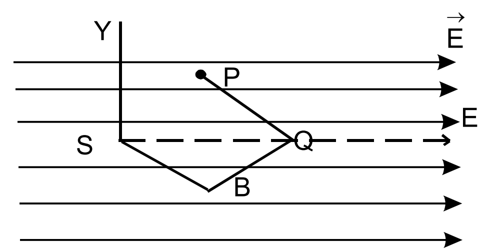
$0 + eV_1 = \frac{1}{2}mv^2 + eV_2$

$\frac{1}{2}mv^2 = e(V_2 - V_1) = eV$

$v = \sqrt{\frac{2eV}{m}}$

[When an electron is acclerated through potential difference 'V' the following formula is generally used $\frac{1}{2}mv^2 = eV$]

Ex.39 A charge q moves along the path PQRS in an electric field E which is directed towards positive X-axis. P, Q , R , S, have the coordinates (a , b , 0) , (2a , 0 ,0) (a , -b , 0) , (0 , 0 ,0) respectively. What is the work done by electric field in this process ?



Sol. $\vec{E} = E\hat{i}$, $\vec{F} = q\vec{E} = q.E\hat{i}$

displacement = $\vec{PS}$

$= (0 - a)\hat{i} + (0 - b)\hat{j} + (0 - 0)\hat{k} = -a\hat{i} - b\hat{j}$

$W = \vec{F}.\vec{PS} = (-a\hat{i} - b\hat{j}).qE\hat{i} = -qEa$

8. MOTION OF A CHARGED PARTICLE IN AN ELECTRIC FIELD :

(i) Charged particle experience force in an electric field.

(ii) Magnitude of force on a charge q in an electric field E is F = qE

(iii) Direction of force on a positive charge is same as direction of electric field while it is opposite to direction of electric field in case of negative charge.

Uniform Electric Field

Case : 1

Initial velocity is zero or in the direction of electric field -

$$F = qE$$

$\Rightarrow$ acceleration $a = \frac{qE}{m}$

$\Rightarrow v = u + at$

Distance travelled in time 't' = $S = ut + \frac{1}{2}at^2$

Case : 2

Initial velocity is perpendicular to electric field -

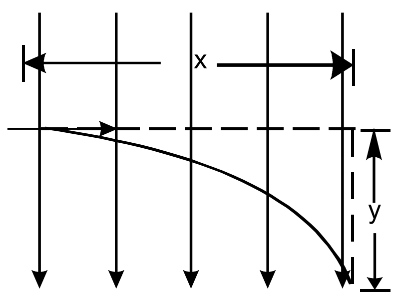
Distance travelled in X direction = ut

Distance travelled in Y direction = $\frac{1}{2}at^2$

where $a = \frac{qE}{m}$

Locus of the path followed -

$$Y = \frac{1}{2}\frac{ax^2}{u^2} \text{ (a parabola)}$$



(iv) Accelerating a charge q through a potential difference V results in

(a) decrease in PE = qV

(b) increase in KE = qV

(v) In a non uniform electric field electron accelerates and translates also.

Example based on Motion of a charged particle in an electric field

Ex.40 An electron is accelerated through 10eV, what will be the velocity acquired by electron.

Sol. We know accelerating charge q through v potential difference increase in K.E. = qV

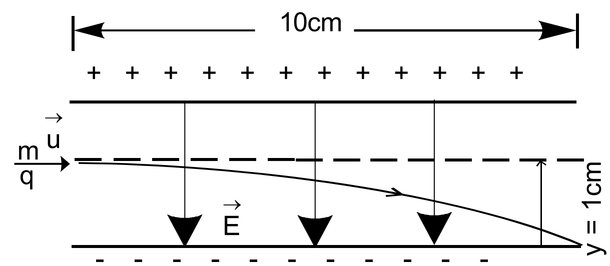
$\Rightarrow \frac{1}{2}mv^2 = 10eV$

$\Rightarrow v = \sqrt{\frac{20\,eV}{m}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-16} \times 10}{9.1 \times 10^{-31}}}$ m/sec

Ex.41 A particle having a charge of 1.6×10^{-19}C enters midway between the plates of a parallel plate capacitor. The initial velocity of particle is parallel to the plates. A potential difference of 300 volts is applied to the capacitor plates. If the length of the capacitor plate is 10cm and they and separated by 2cm. Calculate the greatest initial velocity for which the particle will not be able to come out of the plates. The mass of particle is 12×10^{-24}kg.

Sol. The situation is shown in fig .



Here $E = \frac{\text{Potential difference}}{d}$

$$= \frac{300}{2/100} = 15000 \frac{V}{m}$$

As the particle does not come , its maximum deflection y = 1 cm = 10^{-2} m

We know that $y = \frac{1}{2}\cdot\frac{qE}{m}\left(\frac{x}{u}\right)^2$ or

$u^2 = \frac{1}{2}\cdot\frac{qE}{my}.x^2$

$\frac{1}{2}\frac{(1.6\times10^{-19})(15000)}{(12\times10^{-24})(10^{-2})}\left(\frac{1}{10}\right)^2 = 10^8$

$\therefore$ **u = 10^4 m/s**

Note : When an electron enters normally to electric field, it path becomes parabolic, while in magnetic field path becomes circular

9. ELECTRIC FLUX

(i) It is denoted by 'ϕ'.

(ii) It is a scalar quantity.

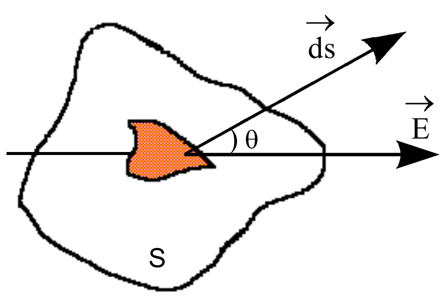
(iii) It is defined as the total number of lines of force passing normally through a curved surface placed in the filed.

(iv) It is given by the dot product of $\vec{E}$ and normal infinitesimal area $\vec{ds}$ integrated over a closed surface-

$$d\phi = \vec{E}.\vec{ds}$$

$$\phi = \oint \vec{E}.\vec{ds} = \oint Eds \cos\theta$$

where θ = angle between electric field and normal to the area



(v) (a) if $\theta = 0$, $\phi = Eds$ (maximum)

(b) if $\theta = 90^0$, ϕ = zero

(vi) Unit : (a) Newton - metre^2 / coulomb.

(b) Volt - meter

(vii) Dimension : $[M L^3 T^{-3} A^{-1}]$

(viii) Flux due to a positive change goes out of the surface while that due to negative change comes into the surface.

(ix) Flux entering is taken as positive while flux leaving is taken as negative

(x) Value of electric flux is independent of shape and size of the surface.

(xi) Flux is associated with all vectors.

(xii) If only a dipole is present in the surface then net flux is zero.

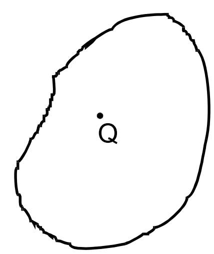
(xiii) Net flux of a surface kept in a uniform electric field is zero.

(xiv) Net flux from a surface is zero does not imply that intensity of electric field is also zero.

10. GAUSS'S LAW

This law states that electric flux ϕ_E through any closed surface is equal to $1/\varepsilon_0$ times the net charge 'q' enclosed by the surface i.e

$$\phi_E = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

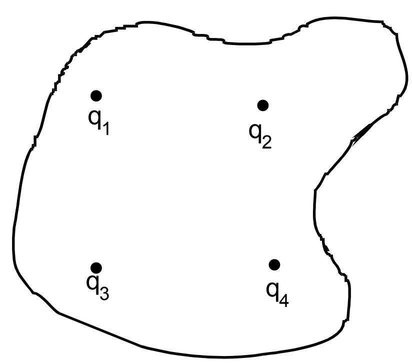


Note :

The closed surface can be hypothetical and then it is called a Gaussian surface.

If the closed surface enclosed a number of charges q_1, q_2 .... q_n etc. then

$$\phi = \oint \vec{E}.\vec{ds} = \frac{\Sigma q}{\varepsilon_0} = \frac{(q_1 + q_2 + ...q_n)}{\varepsilon_0}$$



Flux is -

(i) Independent of distances between charges inside the surface and their distribution.

(ii) Independent of shape , size and nature of surface.

(iii) Dependent on charges enclosed by surface, their nature and on the medium.

(iv) Net flux due to a charge outside the surface will be zero.

(v) If $\Sigma Q = 0$, then $\phi = 0$ but it is not necessary that E = 0

(vi) Gauss law is valid only for the vector fields which obey inverse square law

(vii) Gauss's and coulomb's law are comparable.

Note -

(i) A charge q is placed at the centre of a cube, then ....

(a) Total flux through cube = $\frac{q}{\varepsilon_0}$

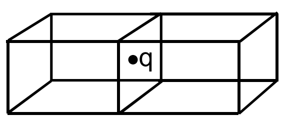
(b) Flux through each surface = $\frac{q}{6\varepsilon_0}$

(ii) A charge q is placed at the centre of a face of a cube ,then total flux through cube

$= \frac{q}{2\varepsilon_0}$

How ? A second cube can be assumed adjacent to the first cube total flux through both cubes

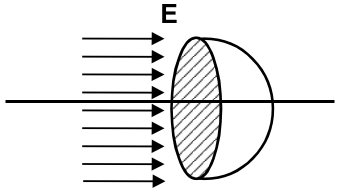
$= \frac{q}{\varepsilon_0}$, So flux through each cube = $\frac{q}{2\varepsilon_0}$



(iii) Now , q is placed at a corner then the flux will be $\frac{q}{8\varepsilon_0}$

Example based on Gauss's Law

Ex.42 A hemispherical surface of radius R is kept in a uniform electric field E such that E is parallel to the axis of hemi-sphere , Net flux from the surface will be -



Sol. $\phi = \oint \vec{E}.\vec{ds} = E \,.\, \pi R^2.$

= (E) (Area of surface perpendicular to E)

$= E.\ \pi R^2.$

Ex.43 A rectangular surface of length 4m and breadth 2m is kept in an electric field of 20 N/c. Angle between the surface and electric field is 30^0. What is flux thought this surface ?

Note : Angle between surface and $\vec{E}$ is given to be 30º. This is not the 'θ' used in our formula 'θ' is the angle between normal to surface and $\vec{E}$. So here $\theta = 90 - 30 = 60º$

Sol. ϕ = EA cos θ = 20 × 8cos60 = 80 V-m

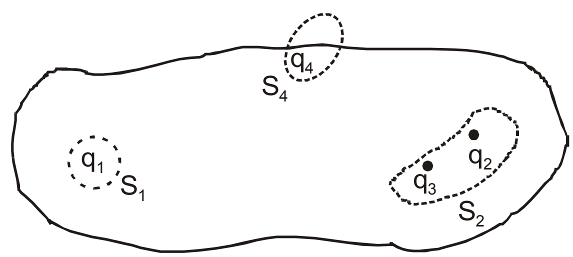
Ex.44 In the following, find out the emerging electric flux through S_1 and S_2 where $[q_1 = 1\mu c\ ,\ q_2 = 2\mu c\ ,\ q_3 = -3\mu c]$

Sol. $\phi_1 = \frac{q_1}{\epsilon_0} = \frac{10^{-6}}{8.85 \times 10^{-12}}$

$= 1.13 \times 10^5$ V.m

$\phi_2 = \frac{q_1 + q_3}{\epsilon_0} = \frac{(2-3)\times 10^{-6}}{8.85 \times 10^{-12}}$

$= -11.3 \times 10^5$ V.m



Ex.45 A charge 'q' is placed at the centre of a cube of side 'a'. If the total flux passing through cube and its each surface be ϕ_1 and ϕ_2 resepctivley then $\phi_1 : \phi_2$ will be -

(A) 1 : 6 (B) 6 : 1

(C) 1 : $6a^2$ (D) $6a^2$: 1

Sol. **(B)** When q is placed at the centre of cube then total flux passing through cube is

$\phi_1 = \frac{q}{\epsilon_0}$

and flux through each surface is $\phi_2 = \frac{q}{6\,\epsilon_0}$

$\therefore\ \phi_1 : \phi_2 = 6 : 1$

Ex.46 If charges q/2 and 2q are placed at the centre of face and at the corner, of a cube. Then total flux through cube will be -

(A) $\frac{q}{2\epsilon_0}$ (B) $\frac{q}{\epsilon_0}$

(C) $\frac{q}{6\epsilon_0}$ (D) $\frac{q}{8\epsilon_0}$

Sol. **(A)** Flux through cube , when q/2 is placed at the centre face , is

$$\phi_1 = \frac{q/2}{2\epsilon_0} = \frac{q}{4\epsilon_0}$$

Flux through cube , which 2q is plaed at the corner of cube , is

$$\phi_2 = \frac{2q}{8\epsilon_0} = \frac{q}{4\epsilon_0},$$

Total flux = $\phi_1 + \phi_2$

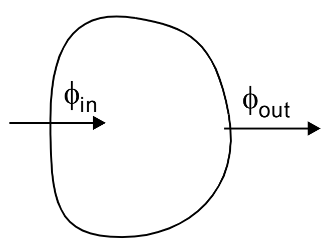
$$= \frac{q}{4\epsilon_0} + \frac{q}{4\epsilon_0} = \frac{1}{2}\frac{q}{\epsilon_0}$$

Ex.47 Flux entering a closed surface is 2000V-m. Flux leaving that surface is 8000 V-m. Find the charge insidesurface.

Sol. Net flux = $\phi_{out} - \phi_{in}$

ϕ = (8000-2000) = 6000V-m

$$\phi = \frac{q}{\varepsilon_0}$$

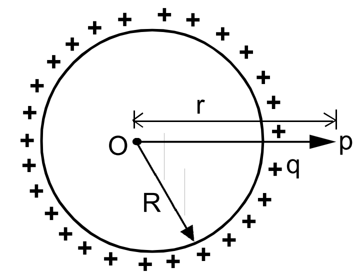


q = (6000) (8.85 × 10^{-12}) = 0.53 μc

11. APPLICATION OF GAUSS'S LAW

11.1 Electric field due to a charged conducting sphere/ Hollow conducting or insulating sphere.

(i) In all the three type of spheres, charge resides only on the outer surface of the sphere in order to remain in minimum potential energy state.



Case: 1 OP = $r \geq R$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{\varepsilon_0}\frac{\sigma R^2}{r^2}\hat{r}$$

(σ = surface charge density)

Case: 2 r = R $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{r}$

Case: 3 r < R $\vec{E} = 0$

i.e. At point interior to a conducting or a hollow sphere, electric field intensity is zero.

(iii) For points outside the sphere , it behaves like all the charge is present at the centre.

(iv) Intensity of electric field is maximum at the surface

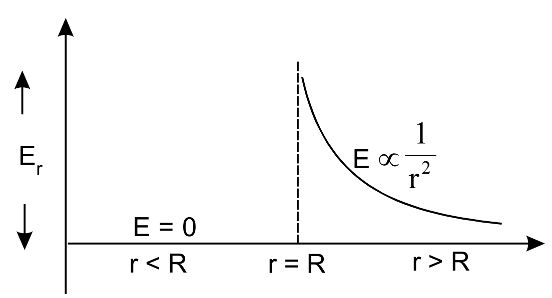
Imp.

(v) Electric field at the surface is always perpendicular to the surface.

(vi) For points, near the surface of the conductor,

$E = \frac{\sigma}{\varepsilon_0}$ perpendicular to the surface

(vii) Graphically ,



Electric potential

Case: 1 r < R

$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Case: 2 r = R

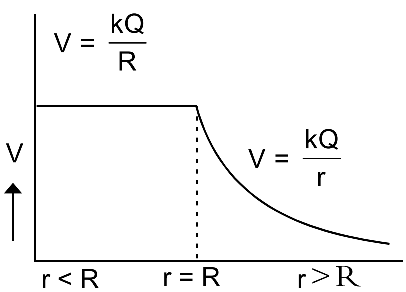
$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Case : 3 r > R

$$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(i) For points interior to a conducting or a hollow sphere, potential is same everywhere and equal to the potential at the surface.

(ii)



(iii) at $r = \infty$, V = 0

Note : Here , we see that $\vec{E}$ inside the sphere is zero but $V \neq 0$. So $\vec{E} = 0$ does not imply V = 0. This presents a good example for it. Similarly V = 0 doesnot imply E = 0

Example based on Application of Gauss's Law

Ex.48 When a charged conductor Q is placed inside a hollow conductor P , in such a way that it touches P , then-

(A) charge will flow from Q to P

(B) Opposite charge will induced on the outer surface of P

(C) whole of the charge of Q will transfer on the internal surface of P

(D) whole of the charge of Q will transfer on the outer surface of P

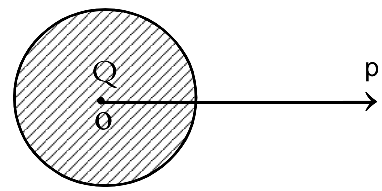
Sol. (D) To keep minimum potential energy whole of the charge of Q will transfer on the outer surface of P.

(B) For a point p at a distance r from centre o.

11.2 Electric field due to solid insulating sphere

A charge given to a solid insulating sphere is distributed equally throughout its volume

Electric Field



Case: 1 r > R (point is outside the sphere)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

Case: 2 r = R (point is at the surface)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r} = E_{max} = E_{surface}$$

Case: 3 r < R (point is inside the sphere)

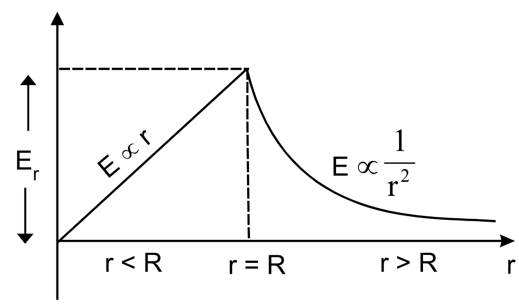
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r\,\hat{r}$$

$$= \frac{\rho r}{3\varepsilon_0}$$

$$E_{in} \propto r$$

at r = 0 , E = 0

(i) Graphically



(ii) Again , for points outside the sphere , it behaves as all the charge is present at the centre

(iii) For points outside , it obeys inverse square law

(iv) Intensity of electric field at infinity is zero.

(v) Intensity at the surface is maximum and is equal to $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

(vi) Again , it is perpendicular to the surface at the surface .

(vii) Intensity is zero at the centre and for points inside the sphere, it is directly proportional to distance of the point from the centre

Electric Potential

Case: 1 $r > R$

$$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Case : 2 $r = R$

$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

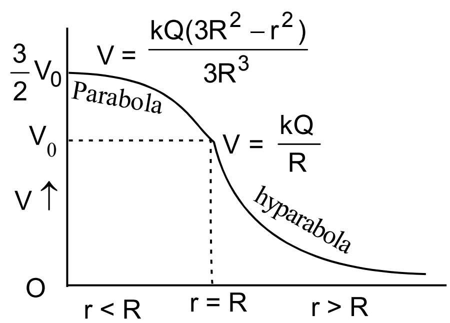
Case : 3 $r < R$

$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q(3R^2 - r^2)}{2R^3}$$

$$V_{centre} = \frac{3}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \qquad \text{(Imp)}$$

$$V_{centre} = 3/2\ V_{surface}$$

(i) Graphically



(ii) Again , $E_{centre} = 0$, but $V_{centre} \neq 0$.

(iii) Electric potential at infinity is zero.

(iv) Electric potential is maximum at the centre

Ex.49 A solid insulating sphere of radius R is given a charge . If inside the sphere at a point the potentail is 1.5 times that of the potential at the surface, this point will be -

(A) At the centre

(B) At distance 3/2R from the centre

(C) potential will be same inside and on the surface of sphere , so given information is inadequate.

(D) Insulating bodies can not be given charge

Sol. (A) Potential at the centre of insulating sphere is given by

$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q(3R^2 - r^2)}{2R^3} \qquad ....(1)$$

and on the surface ,

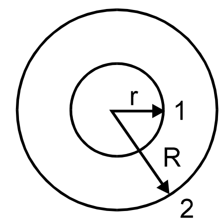
$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \qquad ....(2)$$

given that, $V_{in} = \frac{3}{2}V_{surface}$

$$\Rightarrow \frac{Q(3R^2 - r^2)}{2R^3} = \frac{3}{2}\frac{Q}{R} \Rightarrow r = 0$$

Hence the point will be at the centre.

Ex. 50 Two concentric spheres of radii r & R (r < R) are given the charges q and Q respectively. Find the potential difference between two spheres.



Sol. Potential at the inner sphere = potential due to inner + potential due to outer sphere

$$\therefore V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

(potential at points inside is same everywhere and is equal to potential at the surface).

Potential at outer sphere

V_2 = potential due to inner + potential due to outer sphere

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

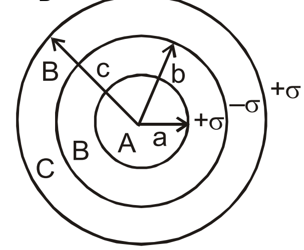
$\therefore$ potential difference = $V_1 - V_2$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \frac{q}{R}\right)$$

$$\therefore \Delta V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Note : Here , we see that 'ΔV' depends only on the charge of inner sphere .

Ex.51 In the following fig, of charged spheres A , B & C whose charge densities are σ , -σ & σ and radii a, b & c respectively what will be the value of V_A & V_B.



Sol. $V_A = \frac{4\pi a^2\sigma}{a} - k\frac{4\pi b^2\sigma}{b} + k\frac{4\pi c^2\sigma}{c}$

$= \frac{\sigma}{\epsilon_0}[a - b + c]$ $[\because \sigma = \frac{q}{A}]$

$V_B = k\left[\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$

$= \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right]$

Ex.52 In the above example if $V_A = V_C$ then what will be relation among a.b & c

Sol. $\because V_A = V_C$ ....(1)

$V_A = \frac{\sigma}{\epsilon_0}[a - b + c]$

$V_C = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - \frac{b^2}{c} + c\right]$

Now from (1) , c = a + b

Ex.53 A solid conducting sphere with a charge Q is placed concentrically inside a second uncharged hollow sphere. Potential difference between the two is V. Now , outer sphere is given a charge of –3q. What will be the potential difference -

Sol. V, since 'ΔV' depends only on the charge of the inner sphere which is not changed.

11.3 Electric field due to infinitely long charge

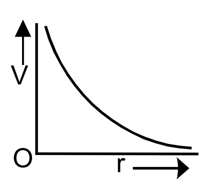
(i) A long wire is given a line charge density λ.

(ii) If wire is positively charged , direction of E will be away from the wire while for a negatively charged wire, direction of $\vec{E}$ will be towards the wire .

(iii) E at point p

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

(iv)



(v) Potential difference between points A (r_1) & $B(r_2) = V_A - V_B = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_2}{r_1}\right)$

(vi) If two charged wires (λ_1) & (λ_2) are kept parallel to each other at a distance 'd', then the force on unit length of any of the wire is :

$$\frac{\lambda_1\lambda_2}{2\varepsilon_0 d}$$

Ex.54 Electric field intensity is proportional to r^{-1} due to

(A) point charge (B) dipole

(C) infinite long charge (D)none of the above

Sol. (3). The electric field intensity due to infinite long charge is given by $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Hence $E \propto \frac{1}{r}$.

Ex.55 A charge q is rotating along a circle of radius r around an infinitely long wire with a line charge density λ. The velocity of charge is

Sol. Here centripetal force $\left(\frac{mv^2}{r}\right)$ is being provided by the electric force of attraction , so

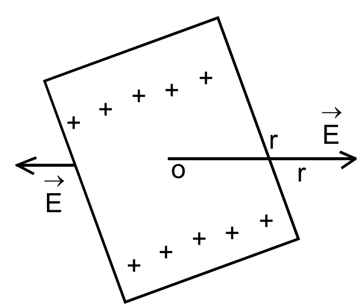
$$\frac{mv^2}{r} = -qE = \frac{q\lambda}{2\pi\varepsilon_0 r}$$

$$\Rightarrow v = \sqrt{\frac{q\lambda}{2\pi\varepsilon_0 m}}$$

[When a charged particle moves along circular orbit in electric field, the following formula is generally used $\frac{mv^2}{r}$ =qE]

11.4 Electric field at a point due to an infinite sheet of charge

(i) If σ = surface charge density. Intensity at points near to the sheet = $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{r}$



(ii) Direction of electric field is perpendicular to the sheet of charge.

(iii) Intensity of electric field does not depend upon the distance of points from the sheet for the points in front of sheet i.e.. There is an equipotential region near the charged sheet.

(iv) Potential difference between two points A & B at distances r_1 & r_2 respectively is

$$V_A - V_B = \frac{\sigma}{2\varepsilon_0}(r_2 - r_1)$$

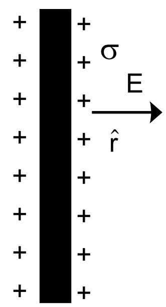
11.5 Electric field due to infinite charged metal sheet

(i) Intensity at points near the sheet

$$= \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{r}$$

where

σ = surface charge density



(ii) $\vec{E}$ is independent of distance of the point from the sheet and also of the area of sheet i.e. There is an equipotential region near the sheet.

(iii) Direction of electric field is perpendicular to the sheet.

(iv) Potential difference between two point A (r_1) and B (r_2) ($r_1 < r_2$) near the sheet is

$$\Delta V = V_A - V_B = \frac{\sigma}{\varepsilon_0}\ (r_2 - r_1)$$

Note : The difference in the cases (D) & (E) is that in (D) σ was only on one side of the sheet while here σ is there on both sides, because it is a metal sheet .

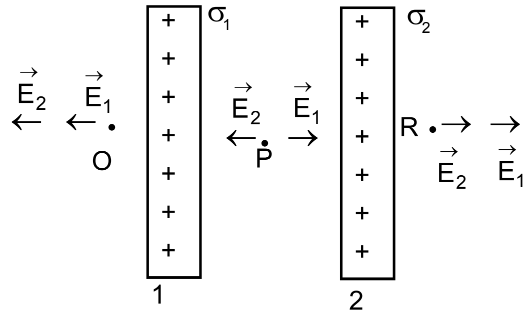
11.6 Electric field due to two infinite parallel plates of charge

(i) Both plates have same type of charge

$$E_O = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

$$E_R = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

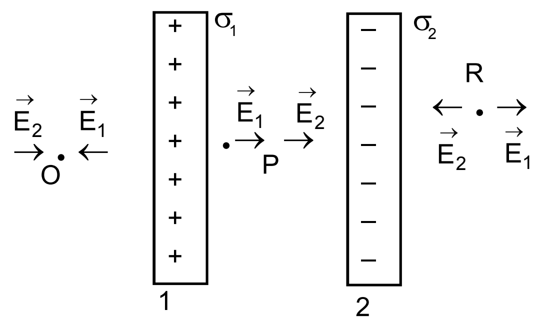


(ii) Two plates have opposite type of charge

$$E_O = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

$$E_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E_R = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$



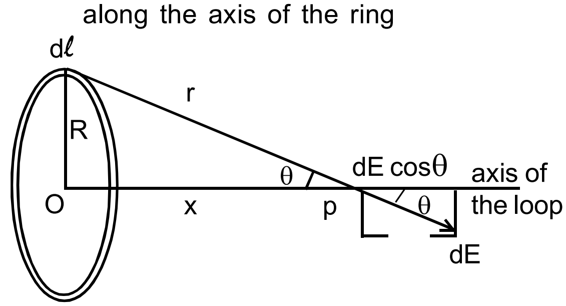
Note : In this case , we will have an uniform electric field between the two plates directed from positive to negative charged plate. Electric field intensity is zero elsewhere.

11.7 Electric field due to charged ring : Q charge is distributed over a ring of radius R.

(i) Intensity of electric field at a distance x from the centre of ring along it's axis -

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)}$$

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q\cos\theta}{r^2}$ and it's direction will be along the axis of the ring



(ii) Intensity will be zero at the centre of the ring.

(iii) Intensity will be maximum at a distance $R/\sqrt{2}$ from the centre and

$$E_{max} = \frac{2}{3\sqrt{3}} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}$$

(iv) Electric potential at a distance x from centre,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{(x^2 + R^2)}}$$

(v) Electric potential will be maximum at the centre and $V_{max} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$

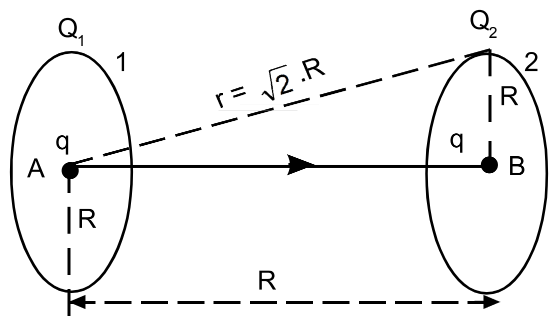
Example based on Electric Field

Ex.56 Two symmetrical rings of radius R each are placed coaxially at a distance R meter. These rings are given the charges Q_1 & Q_2 respectively , uniformly. What will be the work done in moving a charge q from center of one ring to centre of the other.

Sol. Work done = q (V_2 – V_1) potential at the centre of first ring

$$V_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{R} + \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{\sqrt{R^2 + R^2}}$$

$$= \frac{1}{4\pi\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)$$



potential at the centre of second ring

$$V_2 = \frac{1}{4\pi\varepsilon_0 R} \frac{Q_2}{R} + \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{\sqrt{R^2 + R^2}}$$

$$= \frac{1}{4\pi\varepsilon_0 R}\left(Q_2 + \frac{Q_1}{\sqrt{2}}\right)$$

work done = q (V_2 – V_1)

$$= \frac{q}{4\pi\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right)$$

$$W_{1\to 2} = \frac{q}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left(\frac{1}{\sqrt{2}} - 1\right)$$

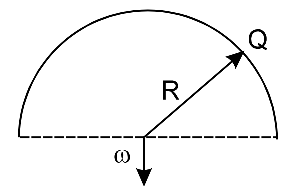
Note : Work done in moving the same charge from second to the first ring will be negative of the work done calculated above i.e.

$$W_{2\to 1} = \frac{q}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left(1 - \frac{1}{\sqrt{2}}\right).$$

11.8 Uniformly charged semi - circular arc

$$E_{centre} = \frac{\lambda}{2\pi\varepsilon_0 R}$$

where ℓ = linear charge density = $\frac{Q}{\pi R}$

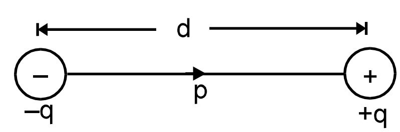


$$V_{centre} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$$

Note : In all of the above discussion , θ is taken to be a positive charge and accordingly the direction of electric field is decided. If θ was negative , all the directions would have been opposite to what they are.

12. ELECTRIC DIPOLE

(i) A system consisting of two equal and opposite charges separated by a small distance is termed an electric dipole.



Example : Na^+Cl^- , $H^+ Cl^-$ etc.

(ii) An isolated atom is not a dipole because centre of positive charge coincides with centre of negative centres. But if atom is placed in an electric field, then the positive and negative centres are displaced relative to each other and atom become a dipole.

(iii) DIPOLE MOMENT : The product of the magnitude of charges and distance between them is called the dipole moment.

(a) This is a vector quantity which is directed from negative to positive charge.

(b) Unit : Coulomb - metre (C-M)

(c) Dimension : $[M^0 L^1 T^1 A^1]$

(d) It is denoted by $\vec{p}$ that is $\vec{p} = q\vec{d}$

12.1 Electric field due to a dipole

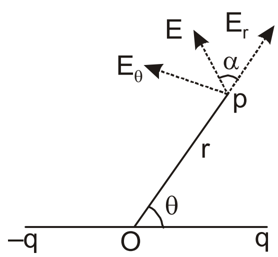
(i) There are two components of electric field at any point

(a) $E_r \rightarrow$ in the direction of $\vec{r}$

(b) $E_\theta \rightarrow$ in the direction perpendicular to $\vec{r}$

$$E_r = \frac{1}{4\pi\varepsilon_0}.\frac{2P\cos\theta}{r^3}$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}.\left(\frac{P\sin\theta}{r^3}\right)$$



(ii) Resultant

$$E = \sqrt{E_r^2 + E_\theta^2} = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

(iii) Angle between the resultant $\vec{E}$ and $\vec{r}$, α is given by $\alpha = \tan^{-1}\left(\frac{E_\theta}{E_r}\right) = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$

(iv) If $\theta = 0$, i.e point is on the axis -

$$E_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{P}{r^3}$$

$\theta = 0$, i .e . along the axis.

(v) If $\theta = 90^o$, i.e. point is on the line bisecting the dipole perpendicularly

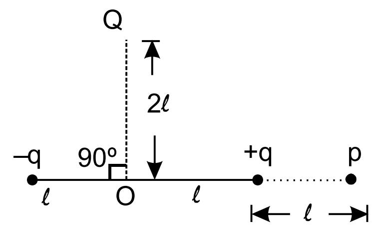
$$E_{equator} = \frac{1}{4\pi\varepsilon_0}.\frac{P}{r^3}$$

(vi) So, $E_{axis} = 2E_{equator}$ (for same r)

(vii) $$E_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{2Pr}{(r^2-\ell^2)^2}$$

$$E_{equator} = \frac{1}{4\pi\varepsilon_0}.\frac{2P}{(r^2+\ell^2)^{3/2}}$$

where $P = q\ .\ (2\ell)$



(viii) $$V = \frac{1}{4\pi\varepsilon_0}.\frac{q(2\ell)\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{P\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}.\frac{\vec{P}.\vec{r}}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{\vec{P}.\hat{r}}{r^3}$$

where θ is the angle between $\vec{P}$ and $\vec{r}$.
V can also be written as

$$V = -\frac{1}{4\pi\varepsilon_0}\vec{P}.\nabla\left(\frac{1}{r}\right) \text{ because } \nabla\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$$

(ix) If $\theta = 0$, $V_{axis} = \dfrac{qd}{4\pi\varepsilon_0 .r^2}$

(x) If $\theta = 90^o$, $V_{equator} = 0$

(xi) Here we see that V = 0 but $E \neq 0$ for points at equator

(xii) Again, if r >> d is not true and $d = 2\ell$,

$$V_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{P}{(r^2-\ell^2)}$$

$$V_{equator} = 0$$

Note :

(i) This is not essential that at a point, where E = 0 , V will also be zero there eg. inside a uniformly charged sphere, E = 0 but V $\neq$ 0

(ii) Also if V = 0 , it is not essential for E to be zero eg. in equatorial position of dipole V = 0, but E $\neq$ 0

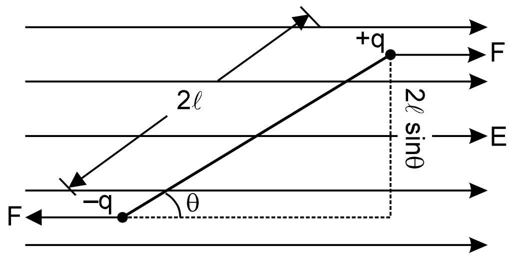
12.2 Electric Dipole In an Electric Field - Uniform Electric Field

(i) When an electric dipole is placed in an uniform electric dipole , A torque acts on it which subjects the dipole to rotatory motion. This τ is given by $\tau = PE\sin\theta$ or

$$\tau = \vec{P}\times\vec{E}$$

(ii) Potential energy of the dipole

$$U = -PE\cos\theta = -\vec{P}.\vec{E}$$



Cases :

(a) If $\theta = 0^o$, i.e. $\vec{P} \parallel \vec{E}$, $\tau = 0$ and $U = -PE$, dipole is in the minimum potential energy state and no torque acting on it and hence it is in the stable equilibrium state.

(b) For $\theta = 180^o$, i.e. $\vec{P}$ and $\vec{E}$ are in opposite direction, then $\tau = 0$ but $U = PE$ which is maximum potential energy state. Although it is in equilibrium but it is not a stable state and a slight perturbation can disturb it.

(c) $\theta = 90^0$, i.e. $\vec{P} \perp \vec{E}$, then

$\tau = PE$ (maximum) and $U = 0$

Note :

(a) There is no net force acting on the dipole in a uniform electric field.

(b) Dipole can only perform rotatory motion.

(c) If dipole is placed in a nonuniform electric field , it performs rotatory as well as translator motion because now a net force also acts on the dipole along with the torque. (important)

12.3 Work done in rotating on electric dipole in an electric field

(i) To rotate the dipole by an angle θ from the state of stable equilibrium $W = PE(1-\cos\theta)$.

(ii) Work done in rotating the dipole from θ_1 to θ_2 in an uniform electric field

$$W = PE(\cos\theta_1 - \cos\theta_2)$$

(iii) Work done in rotating the dipole through 180^0 from stable equilibrium state

$$W = 2PE = 2 \text{ (potential energy)}$$

Example based on Electric Dipole

Ex.57 An electron and a proton are placed at distance of 1Å. What will be dipole moment of so formed dipole

Sol. $p = qd = 1.6 \times 10^{-19} \times 1 \times 10^{-10}$

$= 1.6 \times 10^{-29}$ coulomb metre

Ex.58 E is the intensity of electric field at distance x (axial condition) from the centre of an electric dipole. If the same intensity is at a point distance x' on perpendicular bisector of dipole from its centre , then relation between x & x' will be -

(A) $x' = x$ (B) $x' = x/2$
(C) $x' = x/2^{2/3}$ (D) $x' = x/2^{1/3}$

Sol. **(D)** Given, $E_{axis} = E_{equvatorial}$

$\Rightarrow k\dfrac{2p}{x^3} = k\dfrac{p}{x'^3}$

$\Rightarrow x' = x/2^{1/3}$

Note : All these are valid only if d < < r . otherwise

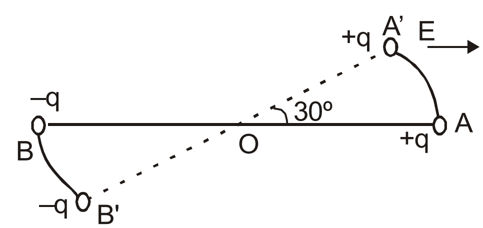
Ex.59 An electric dipole is placed in a uniform electric field $\vec{E}$. What must be the angle between $\vec{E}$ and dipole, so that dipole has minimum potential energy ?

Sol. Zero. $\because U = -PE\sin\theta$

U to be minimum $\theta = 0$

$\therefore U_{min} = 0$

Ex.60 A dipole with dipole moment p is placed in an electric field E. The dipole is displaced from its equilibrium position AB to A'B' as shown in fig. Now what will be the work required , so that the point A' coincids with B.



(A) $\dfrac{2pE(2+\sqrt{3})}{2}$ (B) $\dfrac{pE(2+\sqrt{3})}{2}$

(C) $\dfrac{2pE(2-\sqrt{3})}{2}$ (D) $\dfrac{pE(2-\sqrt{3})}{2}$

Sol.(B) When the dipole is rotated such that it acquires a new position A'B' form position AB then $\theta_1 = 30^0$ Now if dipole is rotated through 180^0 from its position AB then $\theta_2 = 180^0$

Now from figure work done in rotating the dipole form position A'B' so that the point A' coincide with B, is

$$W = pE\,[\cos\theta_1 - \cos\theta_2]$$
$$= PE\,[\cos 30^0 - \cos 180^0]$$
$$= PE\left[\frac{\sqrt{3}}{2} - (-1)\right] = PE\left(\frac{\sqrt{3}+2}{2}\right)$$

13. FORCE ON THE SURFACE OF A CHARGED CONDUCTOR :

(i) If surface charge density on a surface is σ, then electric field intensity at a point near this surface is $\frac{\sigma}{\varepsilon_0}$.

(ii) When a conductor is charged then it's entire surface experiences an outward force perpendicular to the surface.

(iii) The force per unit area of the charged surface is called as the electrical pressure ,

$$P_{electrical\cdot} = \frac{\sigma^2}{2\varepsilon_0}\ \text{N/m}^2.$$

(iv) The direction of this force is perpendicular to the surface.

13.1 Energy associated with the electric field

(i) The energy stored per unit volume around a point in an electric field E is given by

$$U = \frac{1}{2}\varepsilon_0 E^2$$

This is also called energy density

(ii) If in place of vacuum some medium is present then $U = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$.

(iii) For the electric field around a charged conducting sphere $U = \frac{1}{8\pi\varepsilon_0}\cdot\frac{q^2}{R}$

Where q = charge on sphere

R = radius of sphere

(iv) The force of attraction per unit area between plates of parallel plate capacitor is $F = \frac{q^2}{2\varepsilon_0}$

(v) Energy associated with the electric field between plates of parallel plate capacitor is

$$U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad) \text{ where } E = \frac{\sigma}{\varepsilon_0}$$

[These topics can be best studied in the chapter "Capacitance"]

(vi) Work done in charging a parallel plate capacitor is stored as the electric field between plates.

13.2 Drop of a charged liquid -

If n identical drops each having a charge q and radius r coalesce to form a single large drop of radius R and charge Q, then

(a) Charge will be conserved i.e. nq = Q

(b) Volume will be conserved i.e.

$$n\ .\ \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \text{ or } R = n^{1/3}\,r$$

(c) Potential of each small drops = $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$

(d) Potential of large drop = V'

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = V' = n^{2/3}\ V$$

(e) Electric field at surface of small drop = E

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

(d) Electric field at surface of large drop = E'

$$E' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}$$

$$E' = n^{1/3}\ E.$$

Example based on **Forece on drop of a charged liquid**

Ex.61 1000 equal drops of radius 1cm, and charge 1×10^{-6} C are fused to form one bigger drop. The ratio of potential of bigger drop to one smaller drop, and the electric field intensity on the surface of bigger drop will be respectively-

(A) 100 : 1, 9×10^8 V/m

(B) $(10)^{1/3}$: 1, 9×10^8 V/m

(C) $(100)^{1/3}$, 8×10^8 V/m

(D) $(1000)^{2/3}$: 1, 9×10^6 V/m

Sol. **(A)** Let the potential of one smaller drop be V then potential of bigger drop, is

$V' = n^{2/3}\ V$

$\Rightarrow \frac{V'}{V} = n^{2/3}$

$= (1000)^{2/3} = 100$

$\therefore\ V' : V = 100 : 1$

Also let the electric field on the surface of smaller drop be E then electric field on bigger drop is

$E' = n^{1/3}\ E = n^{1/3}\ \frac{kq}{r^2}$

$= (1000)^{1/3}\ \frac{9\times10^9\times1\times10^{-6}}{(1\times10^{-2})^2}$

$= 9 \times 10^8$ V/m

[**Note :** also charge density bigger drop

$\sigma' = \sigma^{1/3} = \left(\frac{q}{4\pi R^2}\right)^{1/3}$]

POINTS TO REMEMBER

(1) The charge density and intensity of electric field is greater at the sharper end, but the electric potential remains same at all the points.

(2) The workdone in carrying a point charge in electric field, does not depend upon the path, because electric field remains conserved.

(3) Potential due to a monopole charge

$$V \propto \frac{1}{x}$$

Potential due to dipole charge

$$V \propto \frac{1}{x^2}$$

Potential due to tetrapole charge

$$V \propto \frac{1}{x^3}$$

(4) If n equal drops of radius r and charge density σ form one big drop, then the charge density of big drop $\sigma' = \sigma^{1/3}$

(5) If equal charge q is placed at points r, 2r, 4r, 8r, ...∞ from a point 'P', then potential at 'P' will be V= 2kq/r

(6) The work done in moving a charge in circular orbit, in a electric field is zero.

(7) If a charged particle of charge q and kinetic energy E, moving about a nucleus of atomic number z, then the least distance between nucleus and charged particle will be-

For least distance of reach,

potential energy = Kinetic energy

$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left(\frac{ze}{r}\right)q = E$

(where r is least distance of reach)

[Note : E is kinetic energy, not the electric field]

$\Rightarrow r = \frac{1}{4\pi\varepsilon_0}\frac{(ze)q}{E}$ m

(8) If + q and –q charges are placed at the ends of a diagonal of a rectangle, of side a & b, then potential difference between the ends of another diagonal will be

$$V = \frac{2kq(a-b)}{ab}$$

(9) A sphere of 1 cm radius, can not be given charge of 1 coulomb, because the electric field intensity at the surface of sphere will be 9×10^{11}. In air the electric field intensity greater than 3×10^6 V/m, ionizes the air, and the charge of sphere starts leaking.

(10) The electric potential of a conductor is a electric state, which ensures the direction of flow charge.

(11) If a positively charged conductor is connected to earth, then the positive charge of conductor will flow to earth and there by the potential of conductor will be zero.

(12) If a negatively charged conductor is connected to earth, then the negative charge (electron) will flow to earth and there by its potential will be zero.

(13) If a charged conductor is placed inside a hollow spherical conductor and the conducters are connected by wire with each

other, the entire charge of charged conductor will come at the outer surface of outer conductor to have minimum potential energy.

(14) If an electron and proton are moving in a uniform electric field, then the electric force acting on them, will be same, but the acceleration of proton will 1/1836 times that of electron, (Because the mass of proton is 1836 times that of electron)

(15) The electric field inside a charged conductor is zero

(16) The electric potential of a charged conductor is same at inner and outer surfaces

(17) The dipole placed in a uniform electric field experience torque, and the net force acting on it is zero.Therefore in uniform electric field the dipole has rotatory motion only not translatory motion.

(18) When electric dipole is placed in non-uniform field it experience torque as well as net force, then by it has rotatory as well as translatory motion.

(19) The electric field due to electric dipole in end side on position on = 2 (electric field in broad side on position)

(20) The potential of earth is zero.

(21) The work done in moving a charged particle does not depend upon the path.

(22) The best conductor of electricity is silver (Ag)

(23) The bubble of soap always inflates, when it is charged (negatively or positively)

(24) The volume of air inside the soap bubble remains constant, in the process of charging.

(25) If two bodies having charges q_1 and $\pm q_2$ are brought in contanct and again separated, then net charge on each of them will be

$$q = \frac{q_1 \pm q_2}{2}$$

(26) The electric field vanishes in a cavity made in a conductor. This is called electrostatic shielding. It implies that the electric instrument can be protected from outside electric fields by placing it in a box made of a good conducting material.

(27) If a charged particle having a charge q and mass m is moving in an electric field between two points having a potential difference of V volts, then the increase in kinetic energy of the body is

$$\frac{1}{2}mv^2 = Vq \text{ or } v = \left(\frac{qV}{m}\right)$$

(28) Electrophorus is used to charge a body by electrostatic induction.

(29) If E = 0 at any point then it is not necessary that the electrostatic potential at that point will also be zero. It may be finite, as in case of the interior point of a uniformly charged conducting sphere, $E = 0$ but $V \neq 0$.

(30) If V = 0 at any point, then it is not necessary that the intensity of electric field at that point will also be zero, as in case of broad side on position of a dipole, $V = 0$ but $E \neq 0$

(31) If a small charged conductor is placed inside another big and hollow charged conductor and both are joined by a wire then the charge flows from smaller conductor to bigger conductor because the potential of smaller conductor is more than that of bigger conductor.

(32) If two like charges are placed at some distance from each other, then the intensity of field will be zero at any point on the line joining the two charges, somewhere between the charges.

(33) If two unlike charges are placed at some distance from each other, then the intensity of field will be zero at any point lying on the line joining the charges but outside the charges. The neutral point is situated on the side of charge of smaller magnitude.

(34) Polar dielectrics are those dielectrics in which the centre of positive charge of a molecule does not coincides with the centre of negative charge and hence they do not show a dipole moment in the absence of electric field. However, they show a dipole moment when they are placed in external field.

(35) When two charged pith balls having charges q_1 and q_2 are suspended from same point with then help of silk threads then considering the equilibrium of any one ball -

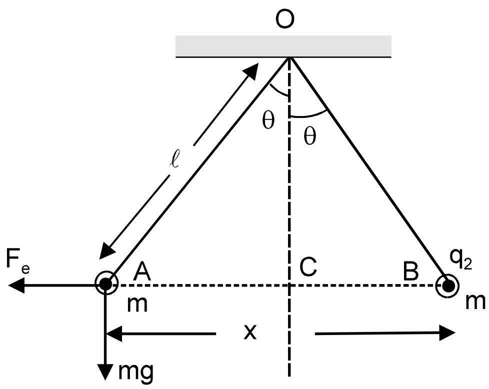
Moment of F_e about 0 = Moment mg about O

$$F_e \times OC = mg\ AC$$

$$\Rightarrow \frac{F_e}{mg} = \tan\theta$$

$$\Rightarrow \frac{q_1 q_2}{4\pi\varepsilon_0 x^2 mg}$$

$$= \tan\theta$$



(36) In the above problem ($x << \ell$) charges on the pith balls are equal then it can be easily proved that

$$x = \left(\frac{q^2 \ell}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

(37) The closed imaginary surface drawn around a charge is called Gaussian surface.

(38) For point charge or spherical distribution of charge, the gaussian surface will be spherical and the electric field will be perpendicular to the surface at all points.

(39) If the flux emerging out of a Gaussian surface is zero then it is not necessary that the intensity of electric field is zero.

(40) Equal amounts of charges can be given to the solid or hollow conducting spheres of equal radius.

(41) With increase in temperature the dielectric constant of liquid increases.