

Electrostatics, deals with the study of charges in rest. These stationary charges occur due to friction of two insulating bodies, therefore it is often called frictional electricity

1. FUNDAMENTAL FORCE OF THE NATURE

- (i) Behind every process occurring in the nature, there is one or the other force acting.
- (ii) Different forces are divided in FOUR parts based on their nature
 - (A) Gravitational force
 - (B) Electro-magnetic force
 - (C) Nuclear force
 - (D) Weak force

(iii) Comparative analysis of forces

| Force | Nature | Range | Rel. Strength |
|------------------|-----------------|------------|---------------|
| Gravitational | Attraction | very large | 1 |
| Electro-Magnetic | Attraction | very large | 10^{36} |
| | or Repulsion | | |
| Nuclear | Attraction | very less | 10^{39} |
| Weak | Unknown | very less | 10^{14} |

Important points :

- (i) Gravitational force is the weakest while nuclear force is the strongest force of the nature
- (ii) Nuclear force does not depend upon charge. It acts equally between proton-proton, proton-neutron and neutron-neutron.
- (iii) There are weak forces acting in β -degradation in radio-activity.
- (iv) A stationary charge produces electric field while a moving charge produces electric as well as magnetic field.
- (v) Moving charge produces electric field as well as magnetic field but does not radiate energy while uniform acceleration.
- (vi) Accelerated charge produces electric field as well as magnetic field and radiates energy.

2. CHARGE

Property of a substance by virtue of which it can repel or attract another charged substance.

Charges are of two types.

- (a) **Positive charge** : Lesser number of electrons than number of protons.

- (b) **Negative charge** : More number of electrons than number of protons

Important Points : Only , electron is responsible for a substance to be charged and not the proton.

2.1 Properties of Charge :

- (i) Like charges repel while unlike charges attract each other.
- (ii) Charge is quantized in nature i.e. The magnitude of charge possessed by different objects is always an integral multiple of charge of electron (or proton) i.e. $q = \pm ne$ where $n = 1, 2, 3, \dots$
- (iii) The minimum possible charge that can exist in nature is the charge of electron which has a magnitude of $e = 1.60207 \times 10^{-19}$ coulomb. This is also known as quantum of charge or fundamental charge.
- (iv) In an isolated system the algebraic sum of total charge remains constant. This is the law of 'Conservation of charge'.

Note : The fact that electric charge is an integral multiple of electronic charge was experimentally proved by Milliken. Unit of charge \rightarrow 1 coulomb = 3×10^9 e.s.u. = $1/10$ e.m.u., in cgs \rightarrow e.s.u. (state coulomb)

Example based on

Charge

Ex.1 Two spheres of the same metal (in all respects) are taken. One is given a positive charge of Q coulomb. and other is given the same but negative charge. Which sphere will have a higher mass.

Sol. Negatively charged sphere will have a higher mass. This is due to increase in number of electron to make it negatively charged.

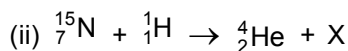
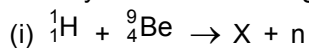
Ex.2 Which of the following charge is not possible:
 (A) $1.6 \times 10^{-18}c$ (B) $1.6 \times 10^{-19}c$
 (C) $1.6 \times 10^{-20}c$ (D) None of these

Sol. (C) $1.6 \times 10^{-20}c$, because this is $1/10$ of electronic charge and hence not an integral multiple.

Ex.3 How many electron are present in 1 coulomb charge.

Sol. $\therefore q = ne$
 $q = 1c$
 $e = 1.6 \times 10^{-19}c$
 $n = ?$
 $n = q/e = 6.25 \times 10^{18}$ electrons.

Ex.4 Identify X in the following nuclear reactions



Sol. (i) Charge on the nucleus of

$$x = Q(\text{H}) + Q(\text{Be}) = 1e + 4e = 5e$$

$$[\because Q(\text{n}) = 0]$$

Therefore X will be Boron

(The atomic no. of B = 5)

(ii) $Q(\text{X}) + Q(\text{He}) = Q(\text{N}) + Q(\text{H})$

$$\Rightarrow Q(\text{X}) = 7e + 1e - 2e = 6e$$

\therefore X is carbon

(ii) This law is valid only for stationary charges and cannot be applied for moving charges.

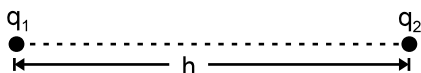
(iii) This law is valid only if the distance between two charges is not less than 10^{-15} m.

(iv) $K = 1$ for air or vacuum,
 $= \infty$ for conductors
 > 1 for any other medium.

| Medium | K |
|------------|----------|
| Vacuum/air | 1 |
| Water | 80 |
| Mica | 8 |
| Glass | 5-10 |
| Metal | ∞ |

3. COULOMB'S LAW

The force of attraction or repulsion between two stationary point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them. This force acts along the line joining the two. If q_1 & q_2 are charges in consideration r , the distance between them and F , the force acting between them



Then, $F \propto q_1 q_2$

$$F \propto 1/r^2$$

$$\therefore F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = k \frac{q_1 q_2}{r^2}, \text{ where } k = \text{constant.}$$

$$k = \frac{1}{4\pi\epsilon_0 K} = \frac{9 \times 10^9}{K} \text{ N}^1 \text{ m}^2 \text{ coulomb}^{-2}$$

where,

ϵ_0 = Electric permittivity of vacuum or air

= $8.85 \times 10^{-12} \text{ coul}^2 \text{ N}^{-1} \text{ m}^{-2}$ and

K = Relative permittivity.

= Dielectric constant

= Specific inductive capacity

[Newton's law for particles is analogous to coulomb's law for rest charges. The difference is that Newton's law gives attraction force while coulomb's law gives attraction as well as repulsion force]

Note:

(i) Coulomb's law is applicable to point charges only. But it can be applied for distributed charges also

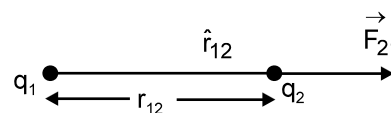
Note : Be aware of k and K

K is dielectric constant and k electrostatic constant

$$k = \frac{1}{4\pi\epsilon_0 K}$$

Direction

Direction of the force acting between two charges depends upon their nature and it is along the line joining two charges.

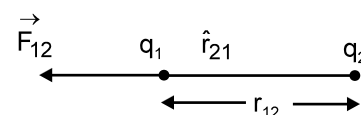


\vec{F}_{21} = force on q_2 due to q_1

$$\vec{F}_{21} = \frac{q_1 q_2 \hat{r}_{12}}{4\pi\epsilon_0 K r_{12}^2} \quad \dots(\text{A})$$

\vec{F}_{12} = Force on q_1 due to q_2

$$\vec{F}_{12} = \frac{q_1 q_2 \hat{r}_{21}}{4\pi\epsilon_0 K r_{12}^2} \hat{r}_{21} \quad \dots(\text{B})$$



Note:

1. $|\hat{r}_{21}| = |\hat{r}_{12}| = 1$ (unit vectors)

2. $\hat{r}_{21} = -\hat{r}_{12} \quad \dots(\text{C})$

3. $\hat{r}_{12} = \hat{r}_{21}$

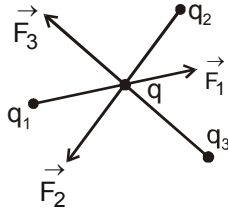
4. Values of q_1 & q_2 are put with sign while using this formula

5. From (A), (B) and (C) $\vec{F}_{12} = -\vec{F}_{21}$

4. PRINCIPLE OF SUPERPOSITION ::

The resultant force acting on a charge due to a group of charges is equal to the vector sum of

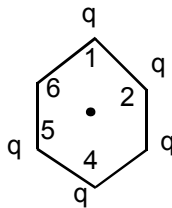
individual forces. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$



Example based on

Principle of superposition

Ex.5 Five equal charges 'q' are placed at five vertices of a regular hexagon. What will be the resultant force on a charge 'Q' placed at the centre of the hexagon given that the distance of a corner from centre is d.



Sol. Suppose, the same charge 'q' was placed at sixth corner also Then

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 = 0$$

(Note that resultant is zero due to symmetry of hexagon. This is applicable for any REGULAR geometry)

$$\begin{aligned} \Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 &= -\vec{F}_6 \\ &= -\frac{qQ}{4\pi\epsilon_0 d^2} \end{aligned}$$

and direction of force will be opposite to \vec{F}_6 .

Ex.6 A point charge q_1 exerts a force F on q_2 . An equal charge q_3 is now kept near q_2 . The resultant force on q_2 due to q_1 will be -

Sol. F . here superposition principle is to be applied carefully. The force on q_2 due to q_1 will remain same although resultant force on

q_2 will change since $\vec{F} = \vec{F}_1 + \vec{F}_3$

Ex.7 Find the ratio of electrostatic and gravitational force acting between two electrons -

Sol. $F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{e \cdot e}{r^2}$; $F_g = G \cdot \frac{m \cdot m}{r^2}$

$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{(4\pi\epsilon_0)G \cdot m^2} \cong 10^{43}$$

Note : 1. $\frac{F_e}{F_g}$ for proton - proton = 10^{36}

2. $\frac{F_e}{F_g}$ for proton electron = 10^{39}

Ex. 8 Force F is acting between two charges. If a sheet of glass ($\epsilon_r = 6$) is placed between the two charges, what will be the force.

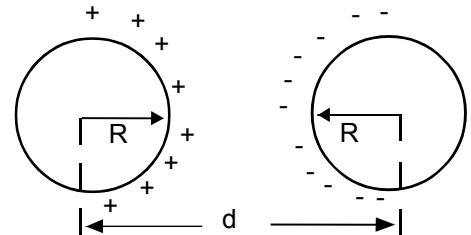
Sol. $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$$F' = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{r^2} \quad 3$$

$$F' = \frac{F}{K} = \frac{F}{6}$$

Note : We can conclude that if there is a metallic medium (conducting) between two charges, force will be zero since $K = \infty$.

Ex.9 Two charged spheres of radius 'R' are kept at a distance 'd' ($d > 2R$). One has a charge +q and the other -q. The force between them will be



(1) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2}$

(2) $> \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2}$

(3) $< \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2}$

(4) None of these

Sol. (2) Redistribution of charge will take place due to mutual attraction and hence effective distance will be less than d.

Note : In the example above, if both had the charge '+q', the answer would have been (3) because now mutual repulsion will result into increase in effective distance.

Ex.10 How should we divide a charge 'Q' to get maximum repulsion between them -

Sol. Let (q) & (Q – q) be the two parts .

$$F = \frac{1}{4\pi\epsilon_0} \frac{q(Q-q)}{r^2}$$

For maximum F

$$\frac{dF}{dq} = 0 \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{Q-2q}{r^2} = 0$$

$$\Rightarrow q = \frac{Q}{2}$$

hence Q should be divided in two equal parts.

Ex.11 $\sqrt{3} \times 10^{-19}$ C and -10^{-6} C are placed at (0, 0, 0) and (1, 1, 1) respectively. Find the force on second in vector form

Sol.
$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$\vec{r}_{12} = (1-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k}$$

$$|\vec{r}_{12}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$$

$$\vec{F}_{21} = \frac{9 \times 10^9 \times \sqrt{3} \times 10^{-19} \times (-10^{-6})}{3} \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$$

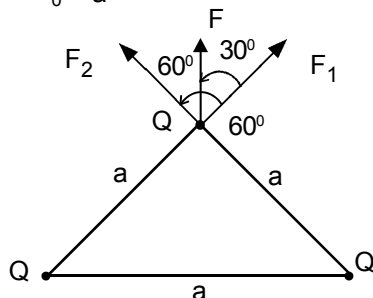
$$= -3 \times 10^{-16} (\hat{i} + \hat{j} + \hat{k}) \text{ Newton.}$$

Ex.12 Three charges (each q.) are placed at the corners of an equilateral triangle. Find out the resultant force on any one charge due to other two.

Sol.
$$F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60^\circ}$$

$$\text{But } F_1 = F_2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2}$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{3} q^2}{a^2}$$



Ex.13 Two charges $1\mu\text{C}$ and $5\mu\text{C}$ are kept at a distance 4cm. The ratio of magnitude of force experienced by first to the second will be -

Sol. 1 : 1

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$= |\vec{F}_{12}| = |\vec{F}_{21}|$$

5. ELECTRIC FIELD ::

A charge produces something called an electric field in the space around it and this electric field exerts a force on any charge placed in it.

Note : The electric field doesnot exert force on source charge.

5.1 Electric field Intensity -

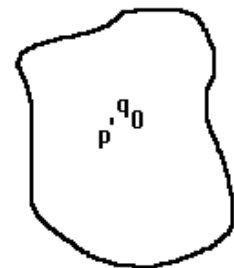
Force experienced by a unit positive charge placed in an electric field at a point is called electric field intensity at that point. It is also known as electric field simply. Let q_0 be the positive test charge placed in an electric field.

If \vec{F} is the force experienced by this charge, then

$$\vec{E} = \text{Electric field intensity} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

(i) Unit : Newton / coulomb or volt/metre

(ii) This is a vector quantity and its direction is the same as force on the positive test charge.



(iii) Since \vec{E} is the force on unit charge, force on charge q is -

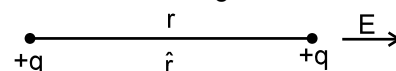
$$\vec{F} = q \vec{E} .$$

(iv) Dimension is $[M^1 L^1 T^{-3} A^{-1}]$

(v) Electric field due to a point charge is

$$\vec{E} = \frac{kq}{r^2} \cdot \hat{r}$$

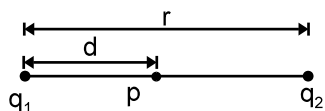
(vi) Direction of electric field due to positive charge is away from charge while direction of electric field due to negative charge is towards the charge.



Special point

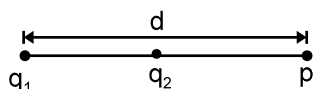
- (a) If q_1 and q_2 are at a distance r and both have the same type of charge, then the distance 'd' of the point from q_1 where electric field is

zero is given by $d = \frac{\sqrt{q_1} r}{(\sqrt{q_1} + \sqrt{q_2})}$. This point will lie between line joining q_1 & q_2 .



- (b) If q_1 and q_2 have opposite charges then distance 'd' of the point 'p' from q_1 where electric field is zero is given by

$$d = \frac{\sqrt{q_1} r}{\sqrt{q_1} - \sqrt{q_2}}, \quad [|q_1| > |q_2|]$$



- (c) Three charges $+Q_1$, $+Q_2$ and q are placed on a straight line. If this system of charges is in equilibrium, charge q should be as given

$$q = -\frac{Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

5.2 Principle of superposition for electric field intensity -

Resultant electric field intensity at a point p due to a number of charges is vector sum of individual electric field intensities $\therefore \vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

Electric field is represented by electric lines of forces

The resultant two electric fields $E_1 + E_2$ is given by $E = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos \theta}$. If the resultant field E , makes an angle with E_1 then

$$\tan \beta = \frac{E_1 \sin \theta}{E_1 + E_2 \cos \theta}$$

5.3 Electric lines of forces :

The electric field in a region can be represented by drawing certain curves known as electric lines of force.

An electric line of force is that imaginary smooth curve drawn in an electric field along which a free isolated unit positive charge moves.

Properties -

- (i) Electric lines of force start from a positive charge and end on a negative charge.
- (ii) No two lines of force can intersect each other. If they do so then at the point of intersection two tangents could be drawn, which gives two directions of electric at the same point, which is impossible.
- (iii) The tangent drawn at any point on line of force gives the direction of force acting on a positive charge placed at that point.
- (iv) These lines have a tendency to contract in length like a stretched elastic string. This actually explains attraction between opposite charges.
- (v) These lines have a tendency to separate from each other in the direction perpendicular to their length. This explains repulsion between like charges.
- (vi) Intensity of electric field is given by the number of electric lines of force in a unit area at that point.
- (vii) Lines of force of a uniform field are parallel and at equal distance.
- (viii) Unit positive charge gives $\frac{4\pi}{K}$ lines in a medium of dielectric constant K .
- (ix) **Important** : Electric lines of force can never enter the conductor, because inside the conductor the intensity of electric field is zero.
- (x) **Important** : Lines of force leave the surface of conductor normally.

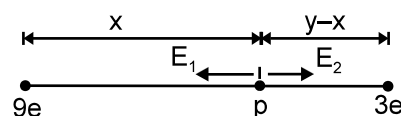
Example based on

Electric field

Ex.14 Charges of $3e$ and $9e$ are placed at a distance r . What is the distance of the point from $9e$ where electric field is zero.

Sol. Putting the values in above formula

$$d = \frac{\sqrt{q} r}{\sqrt{q_1} + \sqrt{q_2}} = \frac{\sqrt{9e} \cdot r}{\sqrt{9e} + \sqrt{3e}} = \frac{\sqrt{3} r}{\sqrt{3} + 1}$$



Systematically : $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{9e}{x^2}$

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{3e}{(r-x)^2}, \quad E_1 = E_2$$

$$\Rightarrow \frac{9e}{x^2} = \frac{3e}{(r-x)^2}$$

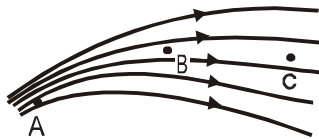
$$\Rightarrow x = \frac{\sqrt{3} r}{\sqrt{3}+1} \quad \text{or} \quad \frac{\sqrt{3} r}{\sqrt{3}-1}$$

$$x = \frac{\sqrt{3} r}{\sqrt{3}-1} \text{ is not possible since } x < r$$

Ex.15 Which is true ?

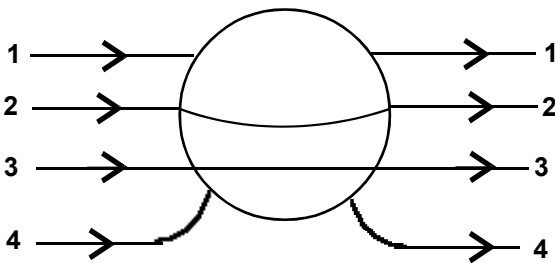
- (A) $E_A < E_B > E_C$ (B) $E_A > E_B > E_C$
 (C) $E_A > E_B < E_C$ (D) $E_A < E_B < E_C$

Sol. (B) Number of electric lines of force in unit area is maximum at A and least at C .



so $E_A > E_B > E_C$.

Ex.16 A metal sphere is placed in an uniform electric field which one is a correct electric line of force-

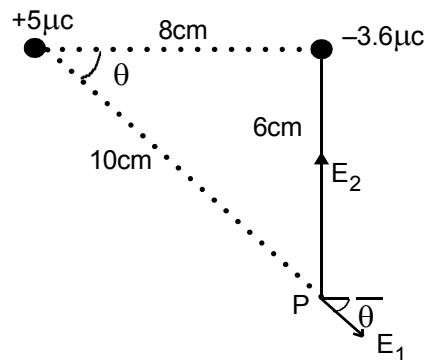


Sol. (4) Only 4 is normal to the conducting surface.

Ex.17 A charge particle is free to move in electric field will it always move along the electric lines of force.

Sol. No. If the particle has its initial velocity = 0, then it will move along the lines of force but if the initial velocity makes some angle with lines of force, the resultant path will not be along the lines of force.

Ex.18 Find \vec{E} at point P.



Sol. E_1 (due to $5\mu\text{C}$) = $9 \times 10^9 \times \frac{5 \times 10^{-6}}{(0.1)^2}$
 $= 4.5 \times 10^6 \text{ N/C}$

$$E_2 \text{ (due to } -3.6 \mu\text{C}) = 9 \times 10^9 \times \frac{3.6 \times 10^{-6}}{(0.06)^2}$$

$$= 9 \times 10^6 \text{ N/C}$$

$$E_x = E_{1x} + E_{2x} = E_1 \cos\theta + 0$$

$$= 3.6 \times 10^5 \text{ N/C}$$

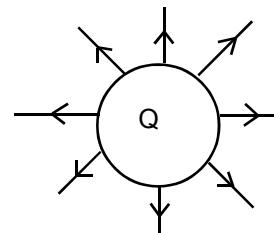
$$E_y = E_{1y} + E_{2y} = -E_1 \sin\theta + E_2$$

$$= 6.3 \times 10^6 \text{ N/C}$$

$$\therefore E = \sqrt{E_x^2 + E_y^2} = 7.3 \times 10^6 \text{ N/C}$$

Note: To avoid mistakes, always take the given quantities in SI units and final answer will also be in the SI units.

Ex.19 The given charge Q is positive or negative ?



Sol. Q is a positive charge because lines are starting from it.

These lines are supposed to terminate at infinity (and not at negative charge). If Q was negative.