

# WORK, POWER, ENERGY

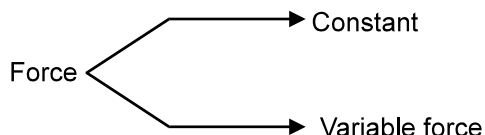
## *Preface*

This is one of the most important chapter in classical mechanics as well as in physics. This chapter needs a crystal clear understanding of kinematical equations and Newton's Laws of motion. You should understand this chapter properly, so that you may able to know the functioning of nature and the laws governing it . After going through the theory notes and few but sufficient solved examples, you will be definitely able to crack the challenging problems given in the exercises. Sound grip on this chapter will certainly help you in electrodynamics , modern physics and thermodynamics etc.

You will get:  
In Chapter Examples  
Solved Examples

## 1. WORK

- Whenever a force acting on a body displaces it, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the body.



### 1.1 WORK DONE BY A CONSTANT FORCE :

If the direction and magnitude of a force applying on a body is constant, the force is said to be constant.



Work done by a constant force,  
 $W = \text{Force} \times \text{component of displacement along force}$   
 $= \text{displacement} \times \text{component of force along displacement}$

If a  $\vec{F}$  force is acting on a body at an angle  $\theta$  to the horizontal and the displacement  $\vec{r}$  is along the horizontal, then the work done will be,  
 $W = (F \cos \theta) r$   
 $= F (r \cos \theta)$

In vector form,  $W = \vec{F} \cdot \vec{r}$

If  $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$  and  $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ , the work done will be,  $W = F_x \cdot x + F_y \cdot y + F_z \cdot z$

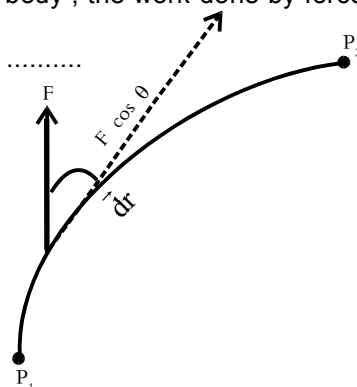
**NOTE :** The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

### 1.2 WORK DONE BY A VARIABLE FORCE ?

If the force applying on a body is changing in its direction or magnitude or both, the force is said to be variable. Suppose a constant force causes displacement in a body from position  $P_1$  to position  $P_2$ . To calculate the work done by the force the path from  $P_1$  to  $P_2$  can be divided into infinitesimal element, each element is so small that during displacement of body through it, the

force is supposed to be constant. If  $d\vec{r}$  be small displacement of body and  $\vec{F}$  be the force applying on the body, the work done by force is

$$dW = \vec{F} \cdot d\vec{r} \dots\dots\dots$$



(1) The total work done in displacing body from

$$P_1 \text{ to } P_2 \text{ is given by, } \int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$

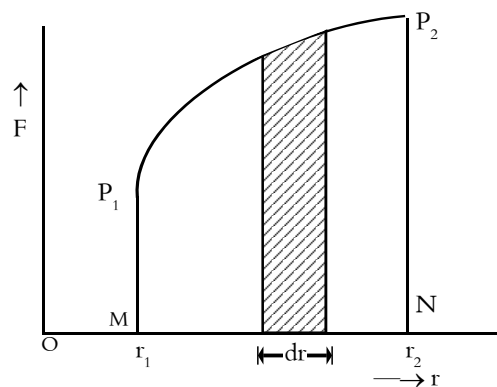
$$\text{or } W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$

If  $\vec{r}_1$  and  $\vec{r}_2$  be the position vectors of the points  $P_1$  and  $P_2$  respectively, the total work done will be -

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

**NOTE :** When we consider a block attached to a spring, the force on the block is  $k$  times the elongation of the spring, where  $k$  is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

### 1.3 Calculation of work done from force displacement graph :



Suppose a body, whose initial position is  $r_1$ , is acted upon by a variable force  $\vec{F}$  and consequently the body acquires its final position  $r_2$ . From position  $r$  to  $r + dr$  or for small displacement  $dr$ , the work done will be  $F \cdot dr$  whose value will be the area of the shaded strip of width  $dr$ .

The work done on the body in displacing it from position  $r_1$  to  $r_2$  will be equal to the sum of areas of all the such strips.

$$\begin{aligned} \text{Thus, total work done, } W &= \sum_{r_1}^{r_2} dW \\ &= \sum_{r_1}^{r_2} F \cdot dr \\ &= \sum_{r_1}^{r_2} F \cdot dr \\ &= \text{Area of } P_1P_2NM \end{aligned}$$

The area between the graph between force and displacement axis is equal to the work done.

**NOTE :** To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between  $F \cos \theta$  and  $r$ .

- (1) Work is a scalar quantity
- (2) The dimensions of work :  $ML^2 T^{-2}$
- (3) **Unit of work :** There are two types of units of work
  - (a) **Absolute unit :** Joule (in M.K.S.), Erg (in C.G.S.) (**NOTE :**  $10^7$  Erg = 1 Joule)
  - (b) **Gravitational unit:** Kilogram - metre (in M.K.S.), Gram-cm (in C.G.S.)

[**NOTE:** 1 kilogram metre = 9.8 joule  
=  $10^5$  gram cm]

#### 1.4 Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero.

**(a) Positive work :** As  $W = \vec{F} \cdot \vec{r} = F r \cos \theta$   
 $\therefore$  When  $\theta$  is acute ( $< 90^\circ$ ),  $\cos \theta$  is positive.  
 Hence work done is positive.

**For example :**

- (i) When a body falls freely under the action of gravity  $\theta = 0^\circ$ ,  $\cos \theta = +1$ , therefore work done by gravity on a body, falling freely is positive.

- (ii) When a gas filled cylinder fitted with a movable piston is allowed to expand, work done by the gas is positive. This is because force due to gaseous pressure and displacement of piston are in same direction.
  - (iii) When a spring is stretched, work done is positive.
- (b) Negative work :** When  $\theta$  is obtuse ( $> 90^\circ$ ),  $\cos \theta$  is negative. Hence work done is negative

**For example :**

- (i) When a body is through up, its motion is opposed by gravity. The angle  $\theta$  between the gravitational force  $\vec{F}$  and displacement  $\vec{r}$  is  $180^\circ$ . As  $\cos \theta = -1$ , therefore, work done by gravity is negative.
- (ii) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction. Hence work done by frictional force is negative. **Note that work done by the applied force is not negative**
- (iii) When a positive charge is moved closer to another positive charge, work done by electrostatic force of repulsion between the charges is negative.

**(c) Zero work :**

When force  $\vec{F}$  or the displacement  $\vec{r}$  or both are zero, work done  $W$ , will be zero. Again when angle  $\theta$  between  $\vec{F}$  and  $\vec{r}$  is  $90^\circ$ , the work done will be zero.

**For example :**

- (i) When we fail to move a heavy stone, however hard we may try, work done by us is zero, because

$$\vec{r} = 0.$$

- (ii) When a coolie carrying some load on his head moves on horizontal platform,  $\theta = 90^\circ$ . Therefore, work done by the coolie is zero.
- (iii) When a body tied to a string is rotated in a circle, the work done by the centripetal force applied along the string is zero. This is because  $\theta = 90^\circ$ .
- (iv) Tension in the string of simple pendulum is always perpendicular to displacement of the bob. Therefore, work done by tension is always zero.

**NOTE :** Another way of expressing negative or positive work is that when energy is transferred to the object work done is positive and when energy is transferred from object the work done is negative and hence the work which is a transfer of energy has same dimension as energy.

Examples based on **Work**

**Ex.1** A body is acted upon by a force

$F = -\hat{i} + 2\hat{j} + 3\hat{k}$ . The work done by the force in displacing it from (0,0,0) to (0,0,4m) will be -

- (A) 12 J (B) 10 J  
(C) 8 J (D) 6 J

**Sol.(A)** Here  $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$  and

$$\vec{d} = (0 - 0)\hat{i} + (0 - 0)\hat{j} + (4 - 0)\hat{k}$$

$$= 4\hat{k}$$

$\therefore$  W (Work done)

$$= \vec{F} \cdot \vec{d} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k}$$

$$= 12 \text{ J}$$

**Ex.2** The work done in pulling a body of mass 5 kg along an inclined plane (angle  $60^\circ$ ) with coefficient of friction 0.2 through 2 m, will be -

- (A) 98.08 J (B) 94.08 J  
(C) 90.08 J (D) 91.08 J

**Sol.(B)** The minimum force with a body is to be pulled up along the inclined plane is  $mg(\sin \theta + \mu \cos \theta)$

$$\text{Work done, } W = \vec{F} \cdot \vec{d}$$

$$= Fd \cos \theta^\circ$$

$$= mg(\sin \theta + \mu \cos \theta) \times d$$

$$= 5 \times 9.8(\sin 60^\circ + 0.2 \cos 60^\circ) \times 2$$

$$= 98.08 \text{ J}$$

**Ex.3** A force  $\vec{F} = (7 - 2x + 3x^2)$  N is applied on a 2 kg mass which displaces it from  $x = 0$  to  $x = 5$  m. Work done in joule is -

- (A) 70 (B) 270  
(C) 35 (D) 135

**Sol.(D)**  $W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2) dx$

$$= [7x]_0^5 - \left[ \frac{2x^2}{2} \right]_0^5 + \left[ \frac{3x^3}{3} \right]_0^5 = 135 \text{ Joule .}$$

**2. POWER ::**

(a) The time rate of doing work is called power

(b) Power =  $\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{x}}{dt}$

In translatory motion :  $P = \vec{F} \cdot \vec{v}$

In rotational motion :  $P = \vec{\tau} \cdot \vec{\omega}$

- (c) It is a scalar quantity  
(d) Unit :

In MKS – J/sec, watt

In CGS – erg/sec,

**(NOTE :** 1KW =  $10^3$  watt, 1HP = 746 watt)

(e) Dimension :  $[M^1 L^2 T^{-3}]$

**NOTE :** Power is the rate at which applied force transfers energy

(A) Power  $\bar{P} = \frac{W}{\Delta t}$  where **W** work is done in  $\Delta t$  time

(B) Instantaneous power  $P = \frac{dW}{dt}$ , it's value may change with time.

Examples based on **Power**

**Ex.4** An automobile of mass  $m$  accelerates from rest. If the engine supplies constant power  $P$ , the velocity at time  $t$  is given by -

(A)  $v = \frac{Pt}{m}$  (B)  $v = \frac{2Pt}{m}$

(C)  $\sqrt{\frac{Pt}{m}}$  (D)  $\sqrt{\frac{2Pt}{m}}$

**Sol.(D)** Given that, power =  $Fv = P = \text{constant}$

or  $m \frac{dv}{dt} v = P$  [as  $F = ma = \frac{mdv}{dt}$ ]

or  $\int v dv = \int \frac{P}{m} dt$

$\Rightarrow \frac{v^2}{2} = \frac{P}{m} t + C_1$

Now as initially, the body is at rest i.e  $v = 0$  at  $t = 0$  so,  $C_1 = 0$

$$\therefore v = \sqrt{\frac{2Pt}{m}} \quad \dots(A)$$

**Ex.5** In the example 4, the position (s) at time (t) is given by -

(A)  $\left(\frac{2Pt}{m}\right)t$                       (B)  $\left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

(C)  $\left(\frac{9P}{8m}\right)^{1/2} t^{1/2}$                       (D)  $\left(\frac{8P}{9m}\right)^{1/2} t$

**Sol.(B)** By definition  $v = \frac{ds}{dt}$  or  $\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$

[From (A)]

$$\Rightarrow \int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$$

$$\Rightarrow s = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2} + C_2$$

Now as  $t = 0$ ,  $s = 0$ , so  $C_2 = 0$

$$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

**Ex.6** A particle moving in a straight line is acted by a force, which works at a constant rate and changes its velocity from  $u$  to  $v$  in passing over a distance  $x$ . The time taken will be -

(A)  $x = \frac{v-u}{v^2+u^2}$                       (B)  $x \left(\frac{v+u}{v^2+u^2}\right)$

(C)  $\frac{3}{2} (x) \left(\frac{v^2-u^2}{v^3-u^3}\right)$                       (D)  $x \left(\frac{v}{u}\right)$

**Sol.(C)**

The force acting on the particle =  $\frac{mdv}{dt}$

Power of the force =  $\left(\frac{mdv}{dt}\right)v = k$  (constant)

$$\Rightarrow m \frac{v^2}{2} = kt + c \quad \dots(1)$$

at  $t = 0$ ,  $v = u \therefore c = \frac{mu^2}{2}$

Now from (1),

$$m \frac{v^2}{2} = kt + \frac{mu^2}{2}$$

$$\Rightarrow \frac{1}{2} m (v^2 - u^2) = kt \quad \dots(2)$$

Again  $\frac{mdv}{dt} v = k$

$$\Rightarrow m.v \frac{dv}{dx} v = k$$

$$\Rightarrow mv^2 dv = kdx$$

Integrating,  $\frac{1}{3} m (v^3 - u^3) = kx \quad \dots(3)$

From (2) and (3),  $t = \frac{3}{2} \left(\frac{v^2 - u^2}{v^3 - u^3}\right) (x)$

### 3. ENERGY ::

- (a) The energy of a body is defined as the capacity of doing work.
- (b) There are various form of energy
  - (i) Mechanical energy    (ii) Chemical energy
  - (iii) Electrical energy    (iv) Magnetic energy
  - (v) Nuclear energy        (vi) Sound energy
  - (vii) Light energy etc
- (c) Energy of system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another.

#### Examples :

Electric energy  $\xrightarrow{\text{Motor}}$  Mechanical energy

Mechanical energy  $\xrightarrow{\text{Generator}}$  Electrical energy

Light energy  $\xrightarrow{\text{Photocell}}$  Electrical energy

Electrical energy  $\xrightarrow{\text{Heater}}$  Heat energy

Electrical energy  $\xrightarrow{\text{Radio}}$  Sound energy

Nuclear energy  $\xrightarrow{\text{Nuclear Reactor}}$  Electrical energy

Chemical energy  $\xrightarrow{\text{Cell}}$  Electrical energy

Electrical energy  $\xrightarrow{\text{Secondary Cell}}$  Chemical energy

Heat energy  $\xrightarrow{\text{Incandescence nt lamp}}$  Light

- (d) Energy is a scalar quantity  
 (e) **Unit** : Its unit is same as that of work or torque.

In **MKS** : Joule , watt sec

In **CGS** : Erg

**NOTE** :  $1\text{eV} = 1.6 \times 10^{-19}$  Joule

$1\text{KWh} = 36 \times 10^5$  Joule

$10^7$  erg = 1 Joule

- (f) **Dimension**:  $[M^1 L^2 T^{-2}]$   
 (g) According to Einstein's mass energy equivalence principle mass and energy are interconvertible i.e. they can be changed into each other

Energy equivalent to mass  $m$  is,  $E = mc^2$

Where ,  $m$  : mass of the particle

$c$  : velocity of light

$E$  : equivalent energy

corresponding to mass  $m$ .

- (h) In mechanics , we do not consider the above fact  
 (i) In mechanics we are concerned with mechanical energy only which is of two type  
 (a) Kinetic energy (b) Potential energy

### 3.1 KINETIC ENERGY

- (a) The energy possessed by a body by virtue of its motion is called kinetic energy  
 (b) If a body of mass  $m$  is moving with velocity  $v$ , its kinetic energy

$$\text{K.E} = \frac{1}{2} mv^2 , \text{ for translatory motion}$$

$$\text{K.E} = \frac{1}{2} I\omega^2 , \text{ for rotational motion}$$

- (c) Kinetic energy is always positive

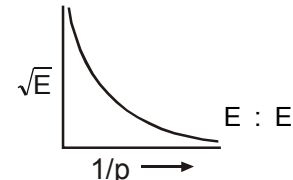
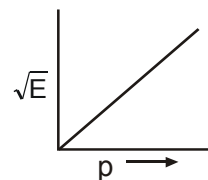
- (d) If linear momentum of body is  $p$ ,  $\text{K.E} = \frac{p^2}{2m}$   
 $= \frac{1}{2} mv^2 =$  for translatory motion.

$$\text{If angular momentum of body is } J, \text{ KE} = \frac{J^2}{2I}$$

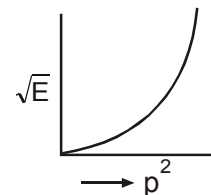
$$= \frac{1}{2} I\omega^2 = \text{for rotational motion}$$

(e)  $p \text{ or } J \propto \sqrt{E}$

$p$  : momentum



$E$  : kinetic energy



- (f) (a) The kinetic energy of a moving body is measured by the amount of work which has been done in bringing the body from the rest position to its present moving position or  
 (b) The kinetic energy of a moving body is measured by the amount of work which the body can do against the external forces before it comes to rest.  
 (g) If a body performs translatory and rotational motion simultaneously, its total kinetic energy  
 $= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

#### 3.1.1 Work Energy Theorem :

- (a) **For translatory motion** :- Work done by all the external forces acting on a body is equal to change in its kinetic energy of translation. Work done by all the external forces = change in K.E. of translation =  $\frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$   
 (b) **For rotational motion** :- Work done by all the external torque acting on a rigid body is equal to change in its rotational kinetic energy. Work done by all the external torque =  $\frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$

**NOTE** : In simple words  $\Delta K = K_f - K_i = W$  in the **work energy theorem** if only energy changed is kinetic energy.

### 3.2 POTENTIAL ENERGY

- (a) The energy which a body has by virtue of its position or configuration in a conservative force field.  
 (b) Potential energy is a relative quantity.

- (c) Potential energy is defined only for conservative force field
- (d) Potential energy of a body at any position in a conservative force field is defined as the work done by an external agent against the action of conservative force in order to shift it from reference point. (PE = 0) to the present position or
- (e) Potential energy of a body in a conservative force field is equal to the work done by the body in moving from its present position to reference position.
- (f) At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.
- (g) Relationship between conservative force field and potential energy (U)

$$\vec{F} = \nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} - \frac{\partial U}{\partial z} \hat{k}$$

eg. (1)  $U = 3x^2 \Rightarrow \vec{F} = -6x \hat{i}$

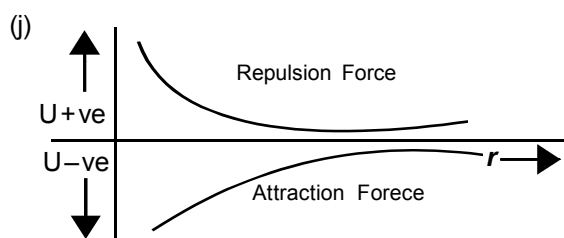
(2)  $U = 2x^2y + 3y^2x + xz^2$

$$\Rightarrow \vec{F} = -(4xy + 3y^2 + z^2) \hat{i} - (2x^2 + 6xy) \hat{j} - (2xz) \hat{k}$$

- (h) If force varies only with one dimension then

$$F = -\frac{dU}{dx} \quad \text{or} \quad U = -\int_{x_1}^{x_2} F dx$$

- (i) Potential energy may be positive or negative
- i) Potential energy is positive, if force field is repulsive in nature
- ii) Potential energy is negative, if force field is attractive in nature



- (k) If  $r \uparrow$  (separation between body and force centre),  $U \uparrow$ , force field is attractive or vice-versa.
- (l) If  $r \uparrow$ ,  $U \downarrow$ , force field is repulsive in nature.

### 3.2.1 DIFFERENT TYPES OF FORCES AND CORRESPONDING POTENTIAL ENERGY (P.E.)

- (a) Gravitational Potential Energy :

- (i) For small distances above or below the earth surface :

Reference point = Earth surface i.e

$$P.E_{\text{surface}} = 0$$

P.E. above the earth surface is positive and below the earth surface is negative and in magnitude it is equal to mgh

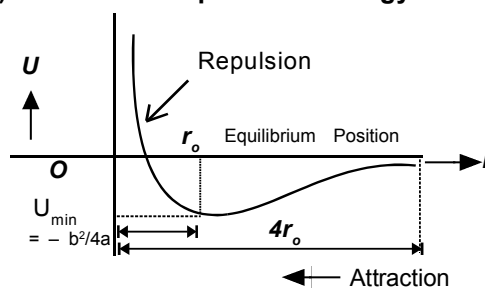
- (ii) For greater distance :

Reference point =  $\infty$  i.e.  $P.E_{\infty} = 0$

$$P.E = -\frac{GMm}{r}, \text{ for } r > R$$

Where R = radius of earth, r = distance of body from the centre of earth, m = mass of body, M = mass of earth

- (b) Electrostatic potential energy :



Reference point =  $\infty$  i.e.  $P.E_{\infty} = 0$

$P.E = \frac{KQ_1Q_2}{r}$  [value of  $Q_1$  and  $Q_2$  are substituted with sign]

- (c) Intermolecular potential energy :

Reference point =  $\infty$  i.e.  $PE_{\infty} = 0$

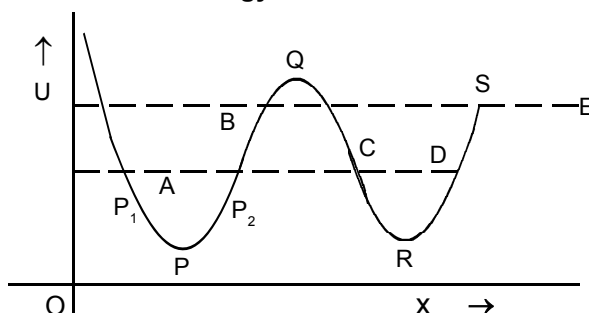
$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}, \quad F = -\frac{dU}{dr} = \frac{6b}{r^7} \left( \frac{2a/b}{r^6} - 1 \right)$$

- (d) Elastic potential energy: Which is associated with the state of compression or

extension of an elastic object  $U(x) = \frac{1}{2} kx^2$

where k = spring constant, x = change in dimension.

### 3.2.2 Potential Energy Curve



- (a) A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve
- (b) Using graph, we can predict the rate of motion of a particle at various positions.
- (c) Force on the particle is  $F_{(x)} = -\frac{dU}{dx}$

**Case : I** On increasing  $x$ , if  $U$  increases, force is in (-)ve  $x$  direction i.e. attraction force.

**Case : II** On increasing  $x$ , if  $U$  decreases, force is in (+)ve  $x$ -direction i.e. repulsion force.

### Different positions of a particle :-

#### Position of equilibrium :-

If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium

$\frac{dU}{dx} = 0$ . Points P, Q, R and S are the states of equilibrium positions.

### 3.2.3 Types of equilibrium :

- (a) **Stable equilibrium** :- When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions:  $-\frac{dU}{dx} = 0$ ,  $\frac{d^2U}{dx^2} = +ve$

- (b) **Unstable Equilibrium** : When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition :  $\frac{dU}{dx} = 0$  potential energy is max

i.e.  $\frac{d^2U}{dx^2} = -ve$

- (c) **Neutral equilibrium** : In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force to acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

Examples based on

## Energy

- Ex.7** The stopping distance for a vehicle of mass  $M$  moving with speed  $v$  along a level road, will be - ( $\mu$  is the coefficient of friction between tyres and the road)

- (A)  $\frac{v^2}{\mu g}$  (B)  $\frac{2v^2}{\mu g}$   
 (C)  $\frac{v^2}{2\mu g}$  (D)  $\frac{v}{\mu g}$

**Sol.(C)** When the vehicle of mass  $m$  is moving with velocity  $v$ , the kinetic energy of the

where  $K = \frac{1}{2} mv^2$  and if  $S$  is the stopping distance, work done by the friction

$$W = FS \cos \theta = \mu MgS \cos 180^\circ = -\mu MgS$$

So by Work-Energy theorem,

$$W = \Delta K = K_f - K_i$$

$$\Rightarrow -\mu MgS = 0 - \frac{1}{2} Mv^2 \Rightarrow S = \frac{v^2}{2\mu g}$$

- Ex.8** The earth circles the sun once a year. How much work would have to be done on the earth to bring it to rest relative to the sun, (ignore the rotation of earth about - its own axis) Given that mass of the earth is  $6 \times 10^{24}$  kg and distance between sun and earth is  $1.5 \times 10^8$  km-

- (A)  $2.7 \times 10^{23} J$  (B)  $2.7 \times 10^{24}$   
 (C)  $1.9 \times 10^{23}$  (D)  $1.9 \times 10^{24}$

**Sol.(A)** As  $T = (2\pi/\omega)$ , so  $\omega = 2\pi/(3.15 \times 10^7)$   
 $= 1.99 \times 10^{-7}$  rad/s

$$\text{Now } v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} \approx 3 \times 10^4 \text{ m/s}$$

Now by work - energy theorem ,

$$W = K_f - K_i$$

$$= 0 - \frac{1}{2} mv^2$$

$$= -\frac{1}{2} \times 6 \times 10^{24} (3 \times 10^4)^2$$

$$= -2.7 \times 10^{33} J$$

Negative sign means force is opposite to the motion.

- Ex.9** A particle of mass  $m$  is moving in a horizontal circle of radius  $r$ , under a centripetal force equal to  $(-k/r^2)$ , where  $k$  is constant. The total energy of the particle is -

- (A)  $\frac{k}{2r}$  (B)  $-\frac{k}{2r}$  (C)  $\frac{k}{r}$  (D)  $-\frac{k}{r}$



**Sol.(B)** As the particle is moving in a circle, so

$$\frac{mv^2}{r} = \frac{k}{r^2} \quad \text{Now K.E} = \frac{1}{2} mv^2 = \frac{k}{2r}$$

$$\text{Now as } F = - \frac{dU}{dr}$$

$$\Rightarrow \text{P.E, } U = - \int_{\infty}^r F dr$$

$$= - \int_{\infty}^r \left( \frac{k}{r^2} \right) dr$$

$$= - \frac{k}{r}$$

$$\text{So total energy} = U + \text{K.E}$$

$$= - \frac{k}{r} + \frac{k}{2r}$$

$$= - \frac{k}{2r}$$

Negative energy means that particle is in bound state .

**Ex.10** The work done by a person in carrying a box of mass 10 kg through a vertical height of 10 m is 4900 J. The mass of the person is -

- (A) 60 kg                      (B) 50 kg  
(C) 40 kg                      (D) 130 kg

**Sol.(C)** Let the mass of the person is m

Work done,  $W = \text{P.E}$  at height  $h$  above the earth surface

$$= (M + m) gh$$

$$\text{or } 4900 = (M + 10) 9.8 \times 10$$

$$\text{or } M = 40 \text{ kg}$$

**Ex.11** A uniform rod of length 4 m and mass 20 kg is lying horizontal on the ground. The work done in keeping it vertical with one of its ends touching the ground, will be -

- (A) 784 J                      (B) 392 J  
(C) 196 J                      (D) 98 J

**Sol.(B)** As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length =  $\ell/2$

$$\text{Work done } W = mgh = mg \frac{\ell}{2}$$

$$= 20 \times 9.8 \times \frac{4}{2}$$

$$= 392 \text{ J}$$

**Ex.12** If  $g$  is acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass  $m$  raised from surface of earth to a height equal to radius  $R$  of the earth is - [ $M$  = mass of earth]

(A)  $\frac{GMm}{2R}$                       (B)  $\frac{GM}{R}$

(C)  $\frac{GMm}{R}$                       (D)  $\frac{GM}{2R}$

**Sol.(A)** We know that the increase in the potential energy

$$\Delta U = GmM \left[ \frac{1}{R} - \frac{1}{R'} \right]$$

according to question  $R' = R + R = 2R$

$$\Delta U = GmM \left[ \frac{1}{R} - \frac{1}{2R} \right] = \frac{GMm}{2R}$$

**Ex.13** A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save ?

- (A) 76%                      (B) 57%  
(C) 38%                      (D) 19 %

**Sol.(C)** In first case,  $W_1 = \frac{1}{2} m(v_1)^2 + mgh$

$$= \frac{1}{2} m(12)^2 + m \times 10 \times 12$$

$$= 72 m + 120 m$$

$$= 192 m$$

and in second case,  $W_2 = mgh$

$$= 120 m$$

The percentage of energy saved

$$= \frac{192m - 120m}{192m} \times 100 = 38\%$$

