Note: The derivative of any function denotes the slope of the respective function or the

Illustrative Examples :

Example 1. If $C = 4x^3 - 3x^2 + 200x$ is a total cost function, find the slope of AC curve and the slope of MC curve when $x = 2$.

Solution :

Total cost function : $C = 4x^3 - 3x^2 + 200x$

Average cost curve : $AC = \dfrac{C}{x} = \dfrac{4x^3 - 3x^2 + 200x}{x} = 4x^2 - 3x + 200$

$\therefore$ The slope of AC curve : $\dfrac{d(AC)}{dx} = \dfrac{d}{dx}(AC)$

$$= \frac{d}{dx}(4x^2 - 3x + 200) = 8x - 3 = 8\,(2) - 3 = 13 > 0, \text{ (positive)}$$

2nd part :

Marginal cost function :

$$\frac{dC}{dx} = \frac{d}{dx}(C) = \frac{d}{dx}(4x^3 - 3x^2 + 200) = 12x^2 - 6x + 200$$

$\therefore$ The slope of MC curve :

$$\frac{d^2C}{dx^2} = \frac{d}{dx}\left(\frac{dC}{dx}\right) = \frac{d}{dx}(MC)$$

$$= \frac{d}{dx}(12x^2 - 6x + 200)$$

$$= 24x - 6 = 24\,(2) - 6 = 42 > 0, \qquad \text{(positive)}$$

Example 2. If the total cost is given by $C = 10 + 2Q + 3Q^2$ where C and Q represent total cost and quantity respectively, find the slope of the average variable cost.

Solution :

Total cost : $C = 10 + 2Q + 3Q^2$

Here total variable cost : $TVC = 2Q + 3Q^2$ $\quad [\because TFC = 10]$

Again, average variable cost :

$$AVC = \frac{TVC}{Q} = \frac{2Q+3Q^2}{Q} = 2+3Q$$

$\therefore$ The slope of AC curve :

$$\frac{d(AVC)}{dQ} = \frac{d}{dQ}(AVC) = \frac{d}{dQ}(2+3Q) = 3 > 0, \quad \text{(positive)}$$

The Shape of Cost Curves

In Figure 20.6, AFC, AVC, ATC, (or AC) and MC curves are shown.

From the above diagram, we observe the following statements,

i) When the level of output increases, AFC decreases and it approaches to ox-axis.

ii) When AVC is minimum, $AVC = MC$

iii) When AC is minimum, $AC = MC$

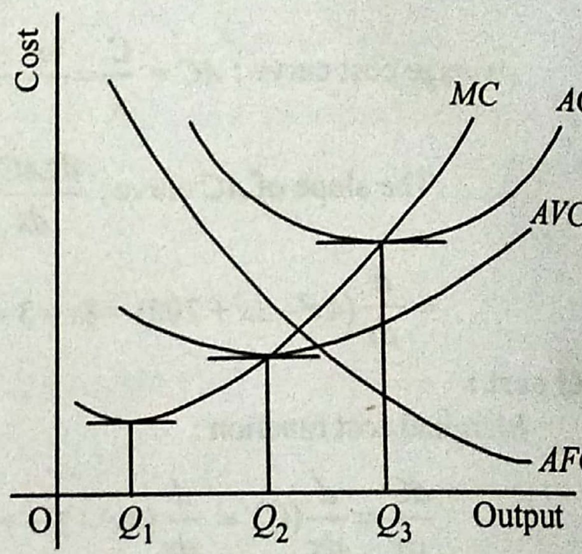

Fig.-20.6 : The shape of cost curves

Conditions for Cost Minimisation

Condition	AC is minimum	MC is minimum	AVC is minimum
First order condition (Necessary condition)	$\frac{d(AC)}{dQ} = 0$	$\frac{d(MC)}{dQ} = 0$	$\frac{d(AVC)}{dQ} = 0$
Second order condition (Sufficient condition)	$\frac{d^2(AC)}{dQ^2} > 0$	$\frac{d^2(MC)}{dQ^2} > 0$	$\frac{d^2(AVC)}{dQ^2} > 0$

Note: By using the above conditions, we can also find the relative extrema values of respective *AC*, *MC* and *AVC* functions.

Illustrative Examples :

Example 1. A manufacturer has a total cost function,

$TC = 100Q - 10Q^2 + Q^3$

Find the output Q which minimises average cost (AC). Also prove that at the level of output $AC = MC$, where MC is the marginal cost. G.U. Exam - 2005

Solution :

Total cost function : $TC = 100Q - 10Q^2 + Q^3$

$$\therefore \quad \text{Average cost : } AC = \frac{TC}{Q} = \frac{100Q - 10Q^2 + Q^3}{Q} = 100 - 10Q + Q^2$$

$$\text{Here, } \frac{d(AC)}{dQ} = \frac{d}{dQ}(AC) = \frac{d}{dQ}(100 - 10Q + Q^2) = -10 + 2Q$$

For minimising AC function, the following two conditions must hold good,

i) $\frac{d(AC)}{dQ} = 0$ ..... First order condition (Necessary condition)

ii) $\frac{d^2(AC)}{dQ^2} > 0$ ..... Second order condition (Sufficient condition)

According to first order condition,

$$\frac{d(AC)}{dQ} = 0$$

$$\Rightarrow \quad -10 + 2Q = 0$$

$$\Rightarrow \quad 2Q = 10$$

$$\therefore \quad Q = \frac{10}{2} = 5$$

Now according to second order condition,

$$\frac{d^2(AC)}{dQ^2} > 0$$

$$\Rightarrow \quad \frac{d}{dQ}\left[\frac{d(AC)}{dQ}\right] > 0$$

$$\Rightarrow \quad \frac{d}{dQ}(-10 + 2Q) > 0$$

$$\therefore \quad 2 > 0$$

Therefore, at $Q = 5$, AC is minimum.

2nd part :

We know that, $AC = 100 - 10Q + Q^2$

Again, marginal cost :

$$MC = \frac{d(TC)}{dQ} = \frac{d}{dQ}(TC)$$

$$= \frac{d}{dQ}(100Q - 10Q^2 + Q^3) = 100 - 20Q + 3Q^2$$

Given that, $AC = MC$

$\Rightarrow \quad 100 - 10Q + Q^2 = 100 - 20Q + 3Q^2$

$\Rightarrow \quad -3Q^2 + Q^2 + 20Q - 10Q - 100 + 100 = 0$

$\Rightarrow \quad -2Q^2 + 10Q = 0$

$\Rightarrow \quad -2Q(Q - 5) = 0$

Here, $-2Q = 0$ and $Q - 5 = 0$

$\therefore \; Q = 0 \qquad \therefore \; Q = 5$

Therefore, at $Q = 5$, average cost is equal to marginal cost.