5 Testing of Hypothesis and Large Sample Test

The theory of testing of hypothesis was initiated by Neyman and Pearson. Procedure for testing the hypothesis is done in following steps.

1. PROCEDURE FOR TESTING OF HYPOTHESIS

(1) Set up the Hypothesis

It is some assumption or statement which may or may not be true about a population. Assumption which we want to test is on the basis of evidence from a random sample. If the hypothesis completely specifies the population, then it is known **as simple** hypothesis otherwise it is known as **composite** hypothesis.

In a sample from a normal population if we specify the mean (μ) as well as variance (σ^2) then it is simple hypothesis.

$$H : \mu = \mu_0 \text{ and } \sigma^2 = \sigma_0^2$$

On the other hand each of the following is composite hypothesis

$$H : \mu = \mu_0 \text{ (Nothing is known about } \sigma^2)$$

$$H : \sigma^2 = \sigma_0^2 \text{ (Nothing is specified about } \mu)$$

For applying tests of significance, we first set up a hypothesis. A hypothesis of no difference is called Null hypothesis and is denoted by H_0 Usually, the null hypothesis is expressed as an equality while the alternative hypothesis is just opposite of null hypothesis and it is always in the form of inequality and is denoted by H_1. It is complementary to the null hypothesis *e.g.* if we want to test the null hypothesis. (H_0) that the population has a specified value say mean μ_0.

$$H_0 : \mu = \mu_0$$

Alternative hypothesis could be

(*i*) $H_1 : \mu \neq \mu_0 \, (\mu < \mu_0 \text{ or } \mu < \mu_0)$

It is known as two-tailed alternative.

(*ii*) $H_1 : \mu > \mu_0$

(*iii*) $H_1 : \mu < \mu_0$

(*ii*) and (*iii*) is known as right tailed and left tailed alternatives respectively.

Type I and Type II Error

In testing of hypothesis we commit two types of errors. The error of rejecting H_0 when H_0 is true is known as Type I Error and error of accepting H_0 when H_0 is false is known as Type II Error.

	Decision from sample	
	Reject H_0	Accept H_0
H_0 True	Wrong (Type I Error)	Correct
H_0 False	Correct	Wrong (Type II error)

P [Reject H_0 when it is true] = P [Type I Error] = α

P (Accept H_0 when it is wrong] = P [Type II Error] = β

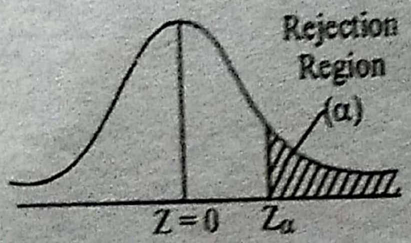
Power of a Test = $1 - \beta$ = P [Accept H_0/H_0 is true] is called power of the test.

(2) Selecting Level of Significance

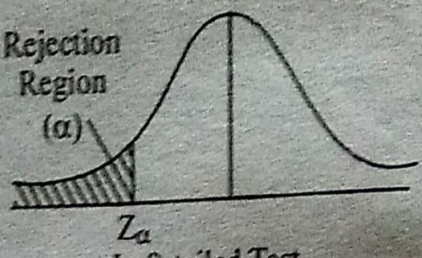
The hypothesis are tested on a pre-determined level of significance. In practice, either 5% level or 1% level is adopted for the purpose. It is denoted by α and $(1 - \alpha)$ is called confidence coefficient.

Critical Region is also called rejection region. This is the area which lead to the rejection of null hypothesis (H_0). If H_1 is single tailed test the critical region lies entirely on the one side. In the right tailed test ($H_1 : \mu > \mu_0$) the critical region lies entirely on the right side of the sampling distributions while for the left tailed test ($H_1 : \mu < \mu_0$) the critical region is entirely in the left tail of the distribution. In case of two tailed alternative hypothesis such as

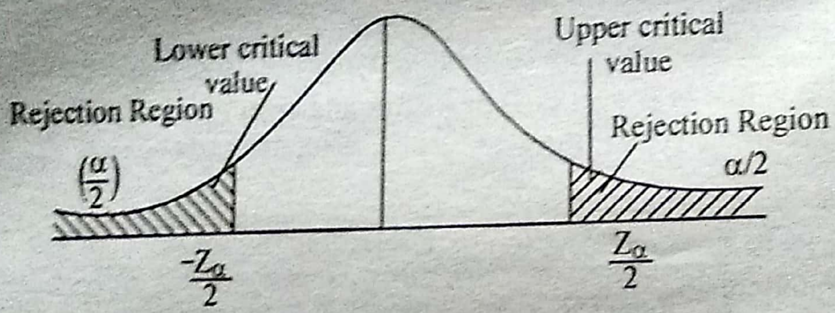
($H_1 : \mu \neq \mu_0$) the critical region is given by the portion of the area lying in both tails of the probability curve.



Right Tailed test



Left-tailed Test



Two tailed Test

(3) Select the suitable test and doing computations

Third step in testing procedure is to construct a test criterion. This involves selecting an appropriate probability distribution that are commonly used in testing procedures are t, F and χ^2. In case of small samples t test, χ^2 test is used. Put the information in the test and do the computations.

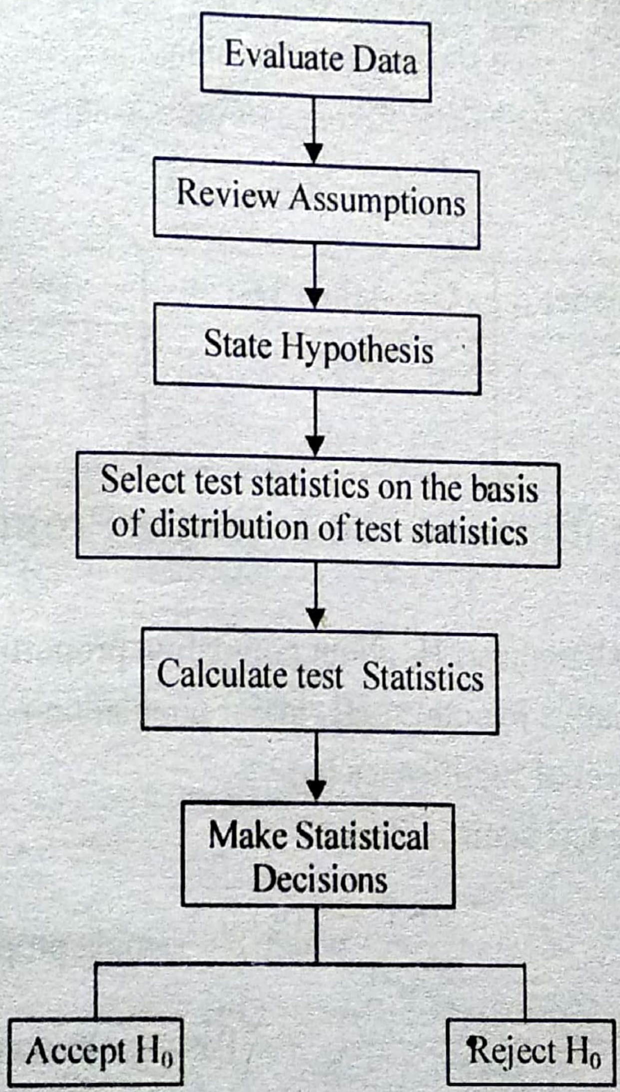
(4) Making Decisions

The decision will depend on whether the computed value of the test criterion falls in the region of rejection or the region of acceptance. If the computed value of the test statistic is greater than the critical value, then we say that it is significant and null hypothesis is rejected at level of the significance.

If the calculated value of the test statistic is less than the tabulated (critical value) we say it is not significant and we accept H_0.

Above steps in the hypothesis testing can be explained by the following flow chart.

Steps in Testing Hypothesis



CONCEPTS USED IN TESTS OF SIGNIFICANCE

(*i*) **Degree of Freedom :** It is abbreviated *d.f.* denotes the extent of independence enjoyed by given set of frequencies. It is usually denoted by v of greek alphabet. If we are given a set of n observed frequencies which are subjected to K independent constraints (restrictions) then

$$d.F = \text{Number of frequencies} - \text{Number of constraints}$$

$$v = n - K$$

(*ii*) **Neyman Pearson Lemma :** The lemma provides the most powerful test of simple hypothesis against a simple alternative hypothesis. Neyman and Pearson introduced a lemma to provide the conditions for getting best critical region of a simple hypothesis against a simple alternative hypothesis.