$$\Rightarrow \vec{S} = \frac{1}{\mu_0} E_0 B_0 \cos^2 (kx-\omega t) (\hat{j} \times \hat{k})$$

$$= \frac{1}{\mu_0} E_0 \times \frac{E_0}{c} \times \cos^2 (kx-\omega t) . \hat{i}$$

(using eqn ⑧)

$$\Rightarrow \vec{S} = \frac{1}{\mu_0 c} \left\{ E_0 \cos (kx-\omega t) \right\}^2 . \hat{i}$$

$$\Rightarrow \vec{S} = c \cdot \epsilon_0 E^2 \hat{i} \longrightarrow (14)$$

$$\Rightarrow \vec{S} = u_{EM} \cdot c . \hat{i} \rightarrow (15)$$

i.e. pointing vector $\vec{S}$ is energy density (u_{EM}) multiplied by the velocity of the wave c.

—x—

Again,

$$\Rightarrow U = \oint \epsilon_0 E_0^2 \cos^2(kx-\omega t) \int dV$$

$$= \epsilon_0 \left(E_0 \cos(kx-\omega t)\right)^2 \cdot V$$

$$\Rightarrow U = \epsilon_0 E^2 \cdot V \quad \rightarrow (11)$$

Thus, Energy density of the EM-wave is,

$$u_{EM} = \frac{U}{V}$$

$$\boxed{\Rightarrow u_{EM} = \epsilon_0 E^2} \quad \rightarrow (12)$$

~~$\Rightarrow u = \frac{B^2}{\mu_0} \rightarrow (13)$~~

$$\boxed{\Rightarrow u_{EM} = \frac{B^2}{\mu_0}} \quad \rightarrow (13) \quad \text{(Using equation (8))}$$

<u>pointing vector:</u>

The pointing vector for monochromatic waves is,

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$= \frac{1}{\mu_0}\left(E_0 \cos(kx-\omega t)\hat{j} \times B_0 \cos(kx-\omega t)\hat{k}\right)$$

We know
Relation between $\vec{E}$ and $\vec{B}$ is,

$$B_0 = \frac{E_0}{c}$$

$$\Rightarrow B_0 = \frac{E_0}{c} \rightarrow (8)$$

Substituting equatn (8) in (7) $\Rightarrow$

$$U = \frac{1}{2}\int_V \left(\frac{E_0^2}{\mu_0 c^2} + \epsilon_0 E_0^2\right) \cos^2(kx-\omega t)\, dv \rightarrow (9)$$

Also,
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow c^2 = \frac{1}{\mu_0 \epsilon_0} \Rightarrow \frac{1}{c^2} = \mu_0 \epsilon_0 \rightarrow (10)$$

Substituting (10) in (9) $\Rightarrow$

$$U = \frac{1}{2}\int_V \left(\frac{E_0^2}{\mu_0} \times \mu_0 \epsilon_0 + \epsilon_0 E_0^2\right) \cos^2(kx-\omega t)\, dv$$

$$= \frac{1}{2}\int_V \left(\epsilon_0 E_0^2 + \epsilon_0 E_0^2\right) \cos^2(kx-\omega t)\, dv$$

$$= \frac{1}{\cancel{2}}\int_V \left(\cancel{2}\, \epsilon_0 E_0^2 \cos^2(kx-\omega t)\right) dv$$

contained in EM field is sumation of energies contained in ~~electrical~~ electric field and magnetic field.

Hence, total energies in EM field is —

$$U = U_E + U_M$$

$$\Rightarrow U = \frac{1}{2}\int_{\text{all space}} \left(\frac{1}{\mu_0}B^2 + \epsilon_0 E^2\right) dv \rightarrow (5)$$

Let us consider the extend of the EM-wave to be within the volume V, hence —

$$U = \frac{1}{2}\int_V \left(\frac{1}{\mu_0}B^2 + \epsilon_0 E^2\right) dv \rightarrow (6)$$

Let us substitute, equaⁿ (1) and (2) in equaⁿ (6) ⇒

$$(6) \Rightarrow \quad U = \frac{1}{2}\int_V \left\{\frac{1}{\mu_0} B_0^2 \cos^2(kx-\omega t) + \epsilon_0 E_0^2 \cos^2(kx-\omega t)\right\} . dv$$

$$\Rightarrow U = \frac{1}{2}\int_V \left(\frac{B_0^2}{\mu_0} + \epsilon_0 E_0^2\right) \cos^2(kx-\omega t) . dv \rightarrow (7)$$

Energy density of Electromagnetic fields:

Let us consider electric and magnetic components of EM-wave are as some plane mono-chromatic plane waves propagating along y- and z-axes ~~repres~~ respectively.

Hence,
$$\vec{E} = E_0 \cos(Kx - \omega t)\,\hat{j} \longrightarrow (1)$$
$$\vec{B} = B_0 \cos(Kx - \omega t)\,\hat{k} \longrightarrow (2)$$

We have already derived the energy stored in electric field and magnetic field:→

Energy stored in an electric field is →

$$U_E = \frac{1}{2}\int_{\text{all space}} \epsilon_0 E^2\, dV \longrightarrow (3)$$

Energy stored in an magnetic field is →

$$U_M = \frac{1}{2}\int_{\text{all space}} \frac{1}{\mu_0} B^2\, dV \longrightarrow (4)$$

Since, EM-wave is combination of Electric and magnetic components hence total energy

~~per~~

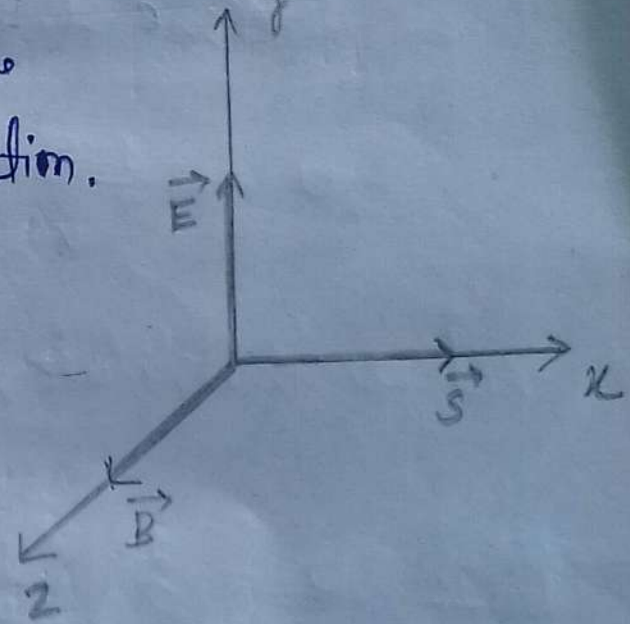
pointing vector:

pointing vector represents the direction of energy transfer per unit area per unit time of an electromagnetic field.

It is denoted by the symbol, $\vec{S}$.

*. The S.I. unit of the pointing vector is watt per metre2 (W/m2).

*. Pointing vector is in the direction of EM wave Propagation.



*. Mathematical formulation of pointing vector is given by →

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \rightarrow (1)$$

In Scalar form,

$$S = \frac{1}{\mu_0} \cdot EB \sin\theta \rightarrow (2)$$