Electromagnetic Wave Propagation through Vacuum:

From Maxwell's equations —

$$\vec{\nabla}.\vec{E} = 0 \rightarrow (i)$$

$$\vec{\nabla}.\vec{B} = 0 \rightarrow (ii)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (iii)$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow (iv)$$

taking curl of equa^n (iii) →

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}.\vec{E} - \nabla 2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow 0 - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) \quad \text{(using equa}^n\text{ (i)}$$

$$\Rightarrow \nabla^2\vec{E} = \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow \nabla^2\vec{E} = \frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}\right) \quad \text{(using equa}^n\text{ (iv)}$$

$$\Rightarrow \nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2 \vec{E}}{\partial t^2} \rightarrow (v)$$

Similarly

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \rightarrow (vi)$$

We know,

$$\nabla^2 \vec{f} = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \vec{f} \rightarrow (vii)$$

Where, v is the velocity.

Comparing (v), (vi) and (vii)

$$\frac{1}{v^2} = \mu_0 \varepsilon_0$$

$$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \rightarrow (viii)$$

Since, in vacuum μ_0 and ε_0 are constant and the values are known to us. Hence, the velocity of EM wave in vacuum is constant.

Substituting the values of μ_0 and ε_0 in equan (viii) for vacuum gives us —

$$V = 3 \times 108 \text{ m/sec} = c$$

i.e. speed of EM-waves in vacuum is constant, c.