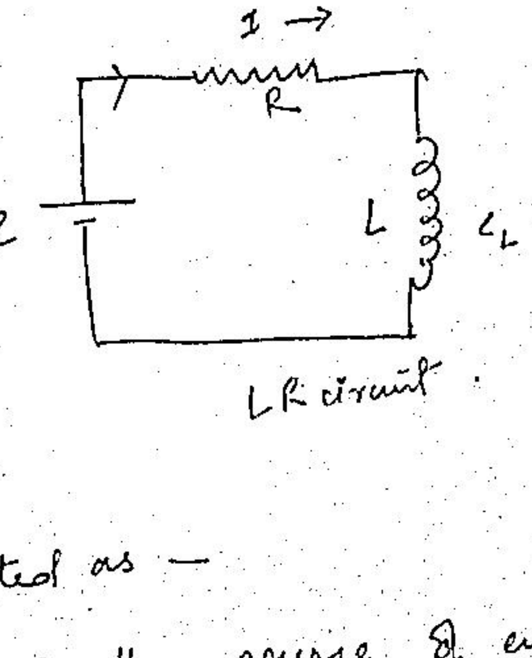
Energy stored in a Magnetic field:

Suppose in a circuit current is increasing, self-inductance will try to decrease this current. Therefore, work must be done to overcome the induced emf and to drive the current against it. This work done is stored up as energy in the system.

To derive a quantitative expression for the storage of the energy in the magnetic field, consider the circuit which shows a source of emf ε connected to a resistance R and inductance L.

From Kirchoff's law,



LR circuit.

$$e = IR + L\frac{dI}{dt}.$$

$$\Rightarrow eI = I^2R + LI\frac{dI}{dt}. \quad \text{(Power)} \longrightarrow (1)$$

Equation (1) can be interpreted as —

(1) If a charge dq passes through the source of emf (battery) in time dt, then the work done by source in time dt is $e\,dq$. The rate of doing work is

$$e\frac{dq}{dt} = eI$$

; eI is the rate at which the source of emf delivers energy to the circuit.

i^2R is the rate at which energy appears as heat in the resistance R. (power dissipated in the resistor)

The energy that doesnot appear as heat must according to law of conservation of energy be stored in the magnetic field.

$\therefore$ $LI\frac{dI}{dt}$ represents the rate at which the energy is stored in the magnetic field. i.e,

$$\frac{dU_B}{dt} = LI\frac{dI}{dt}$$

$$\Rightarrow dU_B = LI\,dI$$

$\therefore$ Energy stored in increasing the current from 0 to I_0 is

$$\boxed{U_B = \int_0^{I_0} LI\,dI = \frac{1}{2}LI_0^2}$$

This is the total stored magnetic energy in an inductance L carrying current I_0.

Again for a solenoid, we have self-inductance

$$L = \mu_0 n^2 A l \qquad \left[B = \mu_0 n I \right.$$

$$L = \frac{B^2 A l}{\mu_0 I^2}$$

$$\boxed{U = \frac{1}{2}LI^2 = \frac{B^2 A l}{2\mu_0}}$$

Alternate derivation

When an electric current is flowing in an inductor, there is energy stored in the magnetic field. Consider an inductor L, the instantaneous power which must be supplied to initiate the current in the inductor is—

$$P = IV = eI \quad [e \rightarrow \text{emf}]$$

$$= L\frac{dI}{dt} I \quad \left[\because e = L\frac{dI}{dt} \text{ Faraday's law} \right]$$

$$= LI\frac{dI}{dt}$$

Since the current is increasing from 0 to I, the total energy (work done) by the battery is given by. Work = Power x time

$$W = \int_0^t P\,dt = \int_0^I LI\frac{dI}{dt}\,dt$$

$$= \int_0^I LI\,dI$$

$$= L\int_0^I I\,dI$$

$$= L\left[\frac{I^2}{2}\right]_0^I = \frac{LI^2}{2}$$

$$\boxed{\text{Energy stored} = \tfrac{1}{2}LI^2}$$