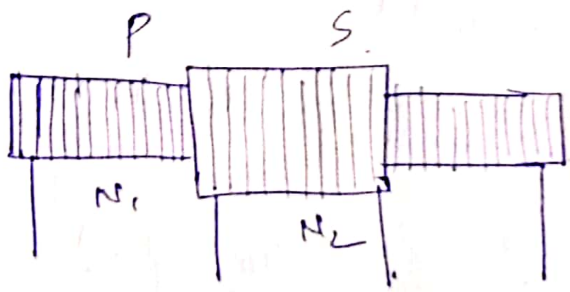
Mutual Induction of Two co-axial solenoids:

In general, the mutual inductance of two coils depends on the geometry of the coils (size, shape, number of turns etc.), the distance between the coils, the relative orientation of the coils and the nature of the material on which the coils are wound.

Let us consider a long solenoid P (primary) of length 'l' and area of cross section A, having N_1 turns. Let a shorter secondary coil S having N_2 turns be wound closely over the central portion of the primary P.



Let i_1 be the current in the primary. The magnitude of the magnetic field inside the primary is in SI unit, given by,

$$B = \mu_0 \frac{N_1}{l} i_1 \quad [\mu_0 n_1 i] \longrightarrow (1)$$

The magnetic flux linked with the primary due to the field B is also linked with the secondary because the secondary is wound closely over the central portion of the primary. Thus the magnetic flux linked with each turn of the secondary is

$$\phi_2 = BA = \mu_0 \frac{N_1}{l} i_1 A. \longrightarrow (2)$$

where A is the area of the cross section of the secondary (although it is almost the same as of the primary.)

The total magnetic flux (flux linkages) through the secondary is,

$$N_2 \phi_2 = N_2 BA = \frac{\mu_0 N_1 N_2 i_1 A}{l} \longrightarrow (3)$$

By definition, the mutual inductance of the two solenoid is,

$$M = \frac{N_2 \phi_2}{i_1} = \frac{\mu_0 N_1 N_2 A}{l} \longrightarrow (4)$$

If n_1 be the number of turns / unit length on the primary, then.

$n_1 = N_1/l$, and we have,

$$\boxed{M = \mu_0 n_1 N_2 A}. \longrightarrow (5)$$

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The coefficient of coupling 'K' of two coils is a measure of the coupling of the coils and is given by

$$K = \sqrt{\frac{M}{L_1 L_2}} \longrightarrow (6)$$

where, L_1 and L_2 are the coefficients of self-induction of the two coils. K is always less than 1

Inductances in series and parallel

(1) Inducatance in series :

When two coils of inductances L_1 and L_2 are connectted in series and a current i is passed through them, the total flux linkages ϕ is the sum of the flux linkages $L_1 i$ and $L_2 i$.

$$\phi = L_1 i + L_2 i$$

or $\phi = Li$ ['L' be the equivalent inductance of the system]

$\therefore Li = L_1 i + L_2 i$

$$\Rightarrow L = L_1 + L_2 \longrightarrow (7)$$

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Equation (7) holds good only when there is a large distance between the two coils so that the current in one doesn't develop a flux linkage in the other.

* When the separation is small between the two coils, there will be a mutual inductance M between them. In this case, the resultant induced emf 'e' in the two coils is the sum of emfs e_1 and e_2 in the respective coils.

$$e = e_1 + e_2$$

$$= \left(-L_1 \frac{\Delta i}{\Delta t} - M\frac{\Delta i}{\Delta t}\right) + \left(-L_2 \frac{\Delta i}{\Delta t} - M\frac{\Delta i}{\Delta t}\right) \longrightarrow (8)$$

Again, we have,

$$e = -L\frac{\Delta i}{\Delta t} \longrightarrow (9)$$

from (8) & (9), we get

$$(-L_1 - M)\frac{\Delta i}{\Delta t} + (-L_2 - M)\frac{\Delta i}{\Delta t} = -L\frac{\Delta i}{\Delta t}$$

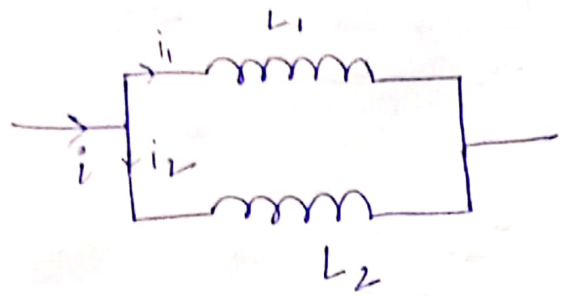
$$\Rightarrow +(L_1 \mp 2M + L_2)\frac{\Delta i}{\Delta t} = +L\frac{\Delta i}{\Delta t}$$

$$\Rightarrow \boxed{L = L_1 + L_2 - 2M} \longrightarrow (9)$$

important.

(2) Inductance in parallel

Suppose two coils L_1 and L_2 are connected in parallel between two points and a current i is divided between them



Then

$$i = i_1 + i_2 \quad \text{[Kirchoff's current law]}.$$

$$\frac{\Delta i}{\Delta t} = \frac{\Delta i_1}{\Delta t} + \frac{\Delta i_2}{\Delta t} \longrightarrow (10)$$

When the currents through the inductances were growing, induced emfs were set up in them. As the potential difference across each coil is the same, the induced emf's must have the same value e. (say).

$$e = -L_1 \frac{\Delta i_1}{\Delta t} = -L_2 \frac{\Delta i_2}{\Delta t} \longrightarrow (11)$$

If L be the equivallent inductance, then

$$e = -L \frac{\Delta i}{\Delta t}. \longrightarrow (12)$$

or, $\frac{e}{L} = -\frac{\Delta i}{\Delta t} = -\left[\frac{\Delta i_1}{\Delta t} + \frac{\Delta i_2}{\Delta t}\right]$

$= \frac{e}{L_1} + \frac{e}{L_2}$ $\left[\because e = L\frac{\Delta i_1}{\Delta t} = L\frac{\Delta i_2}{\Delta t}\right.$

$\Rightarrow \frac{e}{L} = e\left[\frac{1}{L_1} + \frac{1}{L_2}\right]$

$\Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

$\Rightarrow 1/L = \frac{L_1 + L_2}{L_1 L_2}$

$$\Rightarrow \boxed{L = \frac{L_1 L_2}{L_1 + L_2}} \longrightarrow (13)$$

If there is any mutual inductance M between the coils then.

$$\boxed{L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}} \longrightarrow (14)$$

8) A solenoid 3.14 m long and of 0.002 m^2 cross sectional area has 700 turns. Another small solenoid of 50 turns is tightly wound over the longer solenoid. Find the mutual inductance of the two solenoids. ($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})

(Ans: 2.8×10^{-5} H).

Short Questions:

1. Define magnetic flux. Write the SI units of magnetic flux density.
2. Which quantity has 'weber' as its unit?
3. Write the dimensions of magnetic flux.
4. Is magnetic flux a scalar / vector? Magnetic flux density?
5. What is electromagnetic induction? State Faradays law of electromagnetic induction.
6. Write the dimensional formula for induced emf?
7. On which of the following emf induced in a coil doesnot depend?
   (a) No of turns in the coil (b) resistance (c) rate of change of magnetic field.

(8) State Lenz's law of electromagnetic induction.

(9) Does Lenz law violate the law of conservation of energy?

(10) What do you mean by self-induction? What is its SI unit?

(11) Write the dimensional formula of coefficient of self induction.

(12) How does self-inductance of a coil (L) depend on the number of turns (N) of the coil?

(13) What is the effect of metallic core on self-inductance?

(14) What is an inductor? Are the connecting wires in a circuit inductors?

(15) What is an ideal inductor? Why inductors are made of copper?

(16) Name the quantity which in electrical circuits which plays the same role as inertia in mechanics.

(17) What is meant by mutual inductance?

(18) What is the SI unit of mutual Inductance?

Name two factors on which mutual inductance between a pair of coil depends.

(19) Write the dimensional formula for mutual inductance of two coils.