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To continue A^{-1}, find the solution of (inverse of)

$$A = \begin{bmatrix} 2 & 8 \\ 4 & 10 \end{bmatrix}$$

Remember this is 2×2 matrix and slightly tricky.

Next Topic: Simultaneous Equation

What is a Simultaneous Equation system?

It is a system of equations where the number of equations are equal to the number of variables. These variables are also called unknowns.

Basic Question:

What is an equation?

An equation is one where we have '=' sign. Like $5+5=10$ or L.H.S. = R.H.S. Here, L.H.S is $5+5$ and R.H.S is 10

Let us write a Simultaneous equation system consisting of three equations.

$$2x_1 + x_2 + 3x_3 = 15$$
$$x_1 - 2x_2 + 5x_3 = 13$$
$$4x_1 + 3x_2 - x_3 = 11$$

Here, in all the rows, the R.H.S. are constant i.e. absolute numbers or fixed numbers i.e, 15, 13, and 11.

In the columns, we see three Unknowns, also called variables like:

x_1, x_2 and x_3

The numbers multiplied to these Unknowns/variables are constant numbers called co-efficients.

Note that in a simultaneous equation system the number of equations are equal to the number of Unknowns/ Variables.

The solution of a simultaneous equation system is to find out the values of the Unknowns/variables.

There are two methods of finding the solution of a simultaneous equation. One is the direct method by using matrix inversion and the other is the cramer's rule. Let us first discuss the direct method of solution through matrix inversion.

Let us recapitulate A^{-1}. The steps are:

step I: Find the determinant $|A|$

step II: Find the cofactor matrix — cofactor (A)

step III: Find the Adjoint Matrix — Adj (A)

step IV: Find A^{-1} given by the formula

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

We have three equations given above. We also know that the matrix system is written as

$$\begin{bmatrix} \text{coefficient of unknowns} \end{bmatrix} \begin{bmatrix} \text{Unknowns} \end{bmatrix} = \begin{bmatrix} \text{Constants} \end{bmatrix}$$

or

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 5 \\ 4 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 13 \\ 11 \end{bmatrix}$$

Also denoted by

$$AX = C$$

$$\text{or } X = \frac{C}{A}$$

$$\text{or } X = CA^{-1}$$

$$\text{or } \boxed{X = A^{-1}C}$$

This amounts to

$$X = \frac{1}{|A|} \text{Adj } A \cdot C$$

$$\text{or } \boxed{X = \frac{\text{Adj } A}{|A|} C}$$

* To be continued ...