## 5.2 DISPLACEMENT CURRENT

According to Ampere circuital law,

***the line integral of magnetic field $\vec{B}$ around any closed path is equal to $\mu_0$ times the total current threading the closed path, i.e.,***

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I \quad ...(1)$$

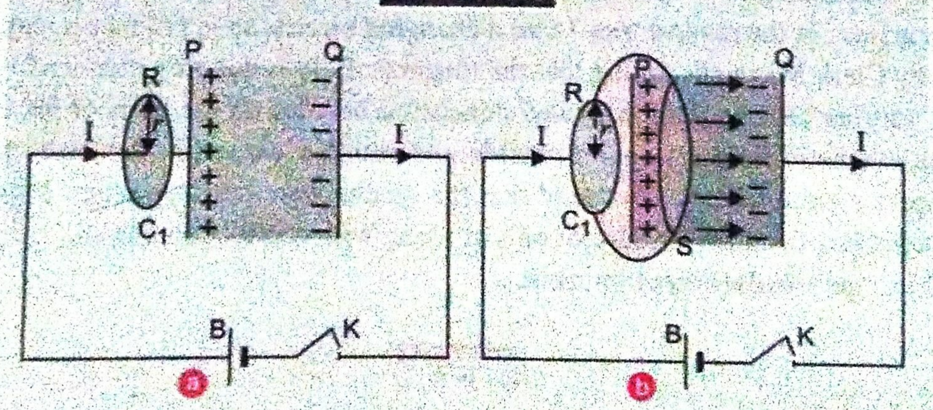
where $I$ is the net current threading the surface bounded by a closed path $C$. Maxwell in 1864 argued that the relation (1) is logically inconsistent. He explained the same with the help of following observations.

Consider a parallel plate capacitor having plates $P$ and $Q$ connected to a battery $B$, through a tapping key $K$. When key $K$ is pressed, the conduction current flows through the connecting wires. The capacitor starts storing charge. As the charge on the capacitor grows, the conduction current in the wires decreases. When the capacitor is fully charged, the conduction current stops flowing in the wires. During charging of capacitor, there is no conduction current between the plates of capacitor. During charging, let at an instant, $I$ be the conduction current in the wires. This current will produce magnetic field around the wires which can be detected by using a compass needle.

Let us find the magnetic field at point $R$ which is at a perpendicular distance $r$ from connecting wire, in a region outside the parallel plate capacitor. For this we consider a plane circular loop $C_1$, of radius $r$, whose centre lies on wire and its plane is perpendicular to the direction of current carrying wire (Fig. 5.1(a)). The magnitude of the magnetic field is same at all points on the loop and is acting tangentially along the circumference of the loop. If $B$ is the magnitude of magnetic field at $R$, then using Ampere's circuital law, for loop $C_1$, we have

$$\oint_{C_1} \vec{B}.d\vec{l} = \oint_{C_1} B\,dl \cos 0° = B\,2\pi r = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r} \quad ...(2)$$

FIGURE 5.1



Now, we consider a different surface, *i.e.*, a tiffin box shaped surface without lid with its circular rim, which has the same boundary as that of loop $C_1$. The box does not touch to the connecting wire and plate *P* of capacitor. The flat circular bottom *S* of the tiffin box lies in between the capacitor plates. Fig. 5.1(b). No conduction current is passing through the tiffin box surface *S*, therefore $I = 0$. On applying Ampere's circuital law to loop $C_1$ of this tiffin box surface, we have

$$\oint_C \vec{B}.\vec{dl} = B\,2\pi r = \mu_0 \times 0 = 0 \quad \text{or} \quad B = 0 \qquad ...(3)$$

From (2) and (3), we note that there is a magnetic field at *R* calculated through one way and no magnetic field at *R*, calculated through another way. Since this contradiction arises from the use of Ampere's circuital law, hence **Ampere's circuital law is logically inconsistent.** Maxwell argued that the above inconsistency of Ampere's circuital law must be due to **something missing.** The missing term must be such that one gets the same magnetic field with whatsoever surface is used. This missing term must be related with a changing electric field which passes through the surface *S* between the plates of capacitor, during charging.

If at the given instant of time, *q* is the charge on the plate of capacitor and *A* is the plate area of capacitor, the magnitude of the electric field between the plates of capacitor is

$$E = \frac{q}{\epsilon_0 A}$$

This field is perpendicular to surface *S*. It has the same magnitude over the area *A* of the capacitor plates and becomes zero outside the capacitor.

The electric flux through the surface *S* is, $\phi_E = \vec{E}.\vec{A} = EA\cos 0^\circ = \frac{1}{\epsilon_0}\frac{q}{A} \times A = \frac{q}{\epsilon_0}$ ...(4)

If $\frac{dq}{dt}$ is the rate of change of charge with time on the plate of the capacitor, then

$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{\epsilon_0}\frac{dq}{dt} \quad \text{or} \quad \frac{dq}{dt} = \epsilon_0 \frac{d\phi_E}{dt}$$

Here, $\frac{dq}{dt}$ = current through surface *S* corresponding to changing electric field = $I_D$, called **Maxwell's displacement current.** Thus,

***displacement current* is that current which comes into play in the region in which the electric field and the electric flux is changing with time.**

This displacement current is given by

$$I_D = \epsilon_0 \frac{d\phi_E}{dt} \qquad ...(5)$$

This displacement current is the missing term in Ampere's circuital law. Maxwell pointed out that for consistency of Ampere's circuital law, there must be displacement current $I_D$ $(= \epsilon_0\, d\phi_E/dt)$ along with conduction current *I* in the closed loop as $(I + I_D)$ has the property of continuity, although individually they may not be continuous.

Maxwell modified Ampere's circuital law in order to make the same logically consistent. He stated Ampere's circuital law to the form,

$$\oint \vec{B}.\vec{dl} = \mu_0 (I + I_D) = \mu_0\left(I + \epsilon_0 \frac{d\phi_E}{dt}\right) \qquad ...(6)$$

This is called **Ampere Maxwell's Law.**