8.1 CURRENT

In electrostatics we have been concerned with stationary electric charges. If a free charge is placed in an electric field it will be acted on by a force and will move in the direction of lines of force. In an isolated metallic conductor, the free electrons present in it are in random motion like the molecules of a gas and the net rate at which electrons pass through any hypothetical plane is zero. If the ends of the conductor are connected to a battery, an electric field E will be set up at every point with in the conductor. This field will act on the electrons and will give them a resultant motion in the direction $-\boldsymbol{E}$. This flow of electrons constitutes an *electric current and the strength of electric current also known as current is defined by the rate at which charge passes a specified point or through any specified surface area* (i.e., $i = q/t$, where q is the net charge passes in time t). In **SI** units its unit is *ampere*, the coulomb per second. It is also the practical unit. If the rate of flow of charge with time is not constant, *i.e.*, the current varies with time, then at any instant t current is defines as

$$i = dq/dt. \quad ..(1)$$

In metals the charge carriers are the free electrons. In electrolytes or in gaseous conductors the positive and negative ions travel in opposite directions. As the positive ions move slower and electrons, the negative ions, move faster, the net motion of charge is due to negatively charged electrons. This type of current is known as *conduction current*. It may be defined as *the steady and orderly movement of free charged particles occurring in a conductor under the influence of an electric field*. Liquids and gases may also undergo hydrodynamic motion, which will produce currents, known as *convection currents*. In this chapter we are not going to discuss convection currents, which are important in atmospheric electricity and in vacuum tubes.

8.2 CURRENT DENSITY, EQUATION OF CONTINUITY

Current is a characteristic of a particular conductor, like the mass, volume, etc. of an object and hence is a *macroscopic quantity*. A related microscopic quantity is the vector **j**, the current density, which is a characteristic of a point inside the conductor rather than of the conductor as a whole. *It is having direction of the flow of positive charge and a magnitude equal to the current per unit area through an infinitesimal area normal to the direction of current flow.*

Thus current density at a given point is the current through a unit normal area at that point. It is measured in amperes/meter^2 or coulomb/meter^2 × sec. If the current is distributed uniformly, the magnitude of **j** for all points on the cross sectional area S is given by $j = i/S$.

The current through any small area Δ **S** (Fig. 8.1) is given by $\Delta i = \mathbf{j}.\Delta\mathbf{S}$. It is same as through the area $\Delta S \cos\theta$ perpendicular to **j**. Thus for large areas, we can write

$$i = \int_S \mathbf{j}.d\mathbf{S}. \qquad ..(2)$$

The value of the integral is independent of the shape of surface taken. *The relation* (2) *shows that the current is the flux associated with* **j** *and is a scalar because the integrand* **j**. d**S** *is a scalar. The arrow often associated with the current in a conductor does not indicate that the current is a vector but shows only the sense of charge flow. Current is not a vector quantity as it does not obey the vector laws.* It remains unchanged if the wire is bent, tied into a knot or distorted. The current is known as *steady or stationary current if* **j** remains constant everywhere.

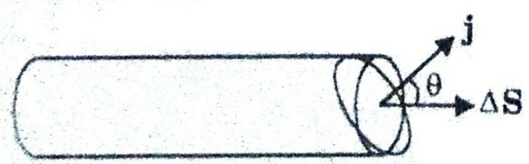

Fig. 8.1. Current through a small element.

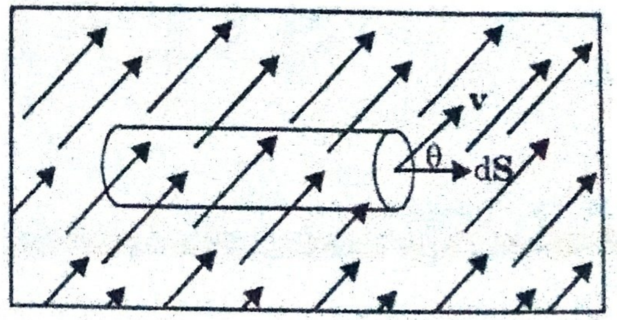

Fig. 8.2. Charge in a cylindrical volume.

To derive an expression for current density, consider a special case in which all the charge carriers have the same charge q and move with the same velocity **v**. In a small time interval Δt, each charge carrier will travel a distance $\mathbf{l} = \mathbf{v}\,\Delta t$. Let us consider a cylinder (or a prism) of cross sectional area $d\mathbf{S}$ and of length **l** in the conductor, Fig. 8.2. The volume of this cylinder will be $d\mathbf{S}\cdot\mathbf{l} = d\mathbf{S}\cdot\mathbf{v}\,\Delta t$. If n is the number of charge carriers per unit volume, then the charge in this cylindrical volume $= qn\,d\mathbf{S}\cdot\mathbf{v}\,\Delta t$. Hence the current or rate of flow of charge $= qn d\mathbf{S}\cdot\mathbf{v}$ and current density $\mathbf{j} = qn\mathbf{v}$.

8.4 CONTINUITY EQUATION AND RELAXATION TIME

Equation of continuity : The current density **j** and the charge density ρ are related at each point through a differential equation. The relation is based on the fact that electric charge can neither be created nor be destroyed and the rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.

$$\therefore \quad i = \int_S \mathbf{j}.d\mathbf{S} = \frac{\partial q}{\partial t} = \frac{\partial}{\partial t}\int_V \rho dV. \qquad ...(18)$$

The minus sign comes here because the direction of $d\mathbf{S}$ is along outward normal and we wish current positive when the net charge is from the outside of V to within. Since we are dealing with a fixed volume V, hence we may put the *RHS* as $\int(\partial\rho/\partial t)\,dV$. Using divergence theorem the surface integral of Eq. (18) may be transformed into a volume integral. Thus we have

$$-\int_V div\,\mathbf{j}\,dV = \int_V \frac{\partial\rho}{\partial t}\,dV \text{ or } \int_V \left(div\ \mathbf{j} + \frac{\partial\rho}{\partial t}\right) dV = 0.$$

This integral must be zero for any arbitrary volume. It is only possible when integrand is zero. Thus,

$$div\,\mathbf{j} + \partial\rho/\partial t = 0. \qquad ...(19)$$

This differential equation is known as the *continuity equation*. If the region does not contain a source or sink of current, $\partial\rho/\partial t = 0$ and hence for *steady current*, we have $div\,\mathbf{j} = 0$.

or

$$\oint_S \mathbf{j} \cdot d\mathbf{S} = 0 \qquad \text{...(20)}$$

for all possible (closed) surfaces S. This means that the charge cannot accumulate anywhere so the *lines of current density are continuous*. Eq. (20) might be called *Gauss's law of currents.*