19/06/21

Note-5

Electromagnetic Induction

Magnetic flux:

If we consider a plane perpendicular to a uniform magnetic field, then the product of the magnitude of the field and the area of the plane is called the "Magnetic flux" 'ϕ' linked with that plane.

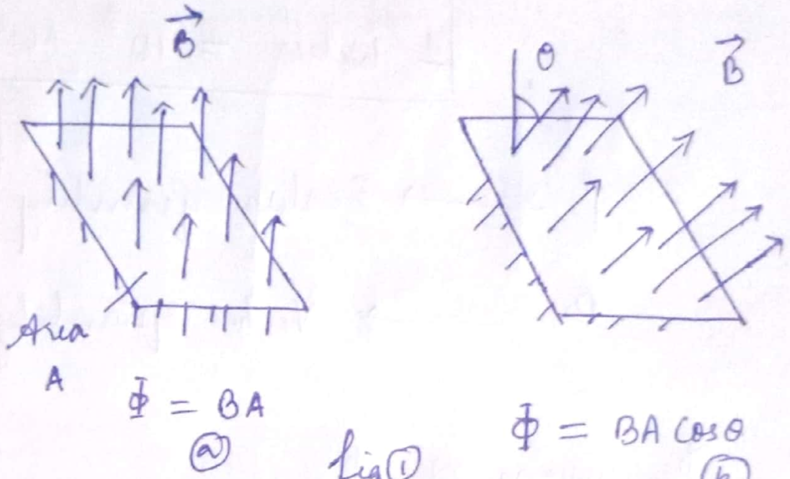

fig (1)

If the magnetic field $\vec{B}$, instead of being perpendicular to the plane, makes an angle θ with the perpendicular to the plane as in fig (1)(b), then the magnetic flux linked with the plane will be equal to the product of the component of the magnetic field $\perp^r$ to the plane and the area of the plane.

Thus, $\phi = (B\cos\theta)A = BA\cos\theta$ ⟶ (1)

ϕ ⟶ +ve if the outward normal to the plane is in the same direction as $\vec{B}$.

ϕ ⟶ −ve if outward normal is opposite to $\vec{B}$.

SI unit of magnetic flux ϕ is "Weber" (Wb)

Since, $B = \frac{\phi}{A}$, the magnetic field is also expressed in webers/metre^2 ($Wb-m^{-2}$)

Thus, magnetic field induction B is also called the magnetic flux density.

The CGS unit of ϕ is "Maxwell"

$$\boxed{1 \text{ Weber} = 10^8 \text{ Maxwell}}$$

$\phi \longrightarrow$ Scalar quantity

$B \longrightarrow$ Vector quantity.

Dimensions of ϕ:

We know the formular,

$F = BiL$ (force on a current-carrying conductor of length L in a magnetic field B),

$$B = \frac{F}{iL}$$

$$\therefore \phi = BA \quad [\text{from eq}^n \text{ ①}]$$

$$= \frac{FA}{iL}$$

Dimension, $[\phi] = \frac{[MLT^{-2}][L^2]}{[A][L]} = [ML^2T^{-2}A^{-1}]$

We can represent a magnetic field by magnetic lines of force. If we draw limited lines of force so that in a magnetic field, $B = 1\ Wb\ m^{-2}$, only one line of force passes / m^2 through an area perpendicular to B or in a field $B = 2\ Wb/m^2$ only two lines of force passes / m^2 perpendicular to B and so on, then these lines of forces are called <u>lines of flux</u>.

"In a magnetic field, the number of lines of flux passing per meter^2 through an area perpendicular to the field is equal to the magnetic flux linked with that plane."

Electromagnetic Induction

In 1831, Faraday discovered that whenever the number of magnetic lines of force, or magnetic flux passing through a circuit changes, an e.m.f (electromotive force) is produced in the circuit. If the circuit is closed, current flows through the circuit. The emf and current so produced are called "<u>induced emf</u>" and "<u>induced current</u>" and last only till the magnetic flux is changing. This phenomenon is known as <u>electromagnetic Induction</u>.

Magnetic flux in a circuit may change by —

1. By moving a magnet relative to the circuit.
2. By changing current in a neighbouring circuit.
3. By changing current in the same circuit.
4. By rotating a coil in a magnetic field.

Example:

(1)

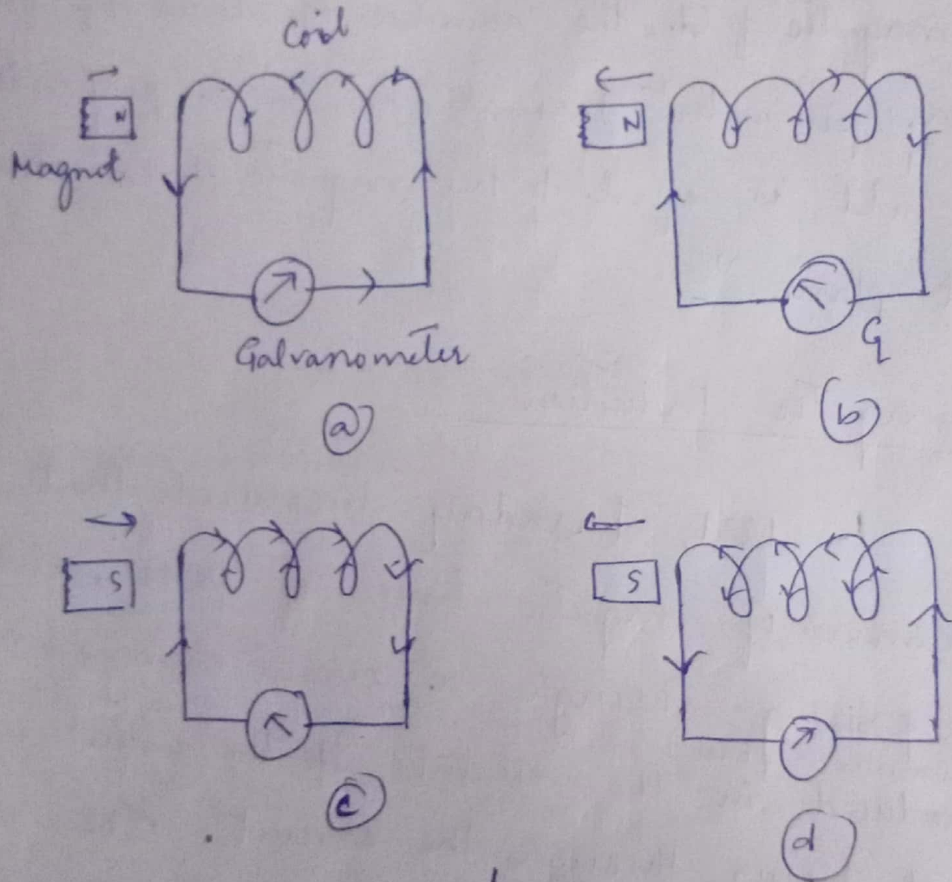

Figure 2

In figure (2) are shown a magnet and a coil connected to a galvanometer. When the magnet is quickly moved towards the coil with the North pole (N) pointing the coil (figure (a)), the galvanometer deflects while the magnet is moving. This indicates a momentary current in the coil.

Now when the magnet is moved away (figure ⑥), the galvanometer again deflects, but in the opposite direction, which means current in the coil is in the opposite direction.

If the experiment is repeated with the south pole of the magnet facing the coil, the deflections are reversed. The faster the motion of the magnet, larger are the deflections.

If the magnet is kept stationary and the coil is moved towards or away from the magnet, even then there is deflection in the galvano-meter. This shows that the current in the coil is produced due to relative motion between the coil and the magnet, it does not matter whether the coil is moving or the magnet is moving.

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~~Faraday's laws of Electromagnetic Induction~~

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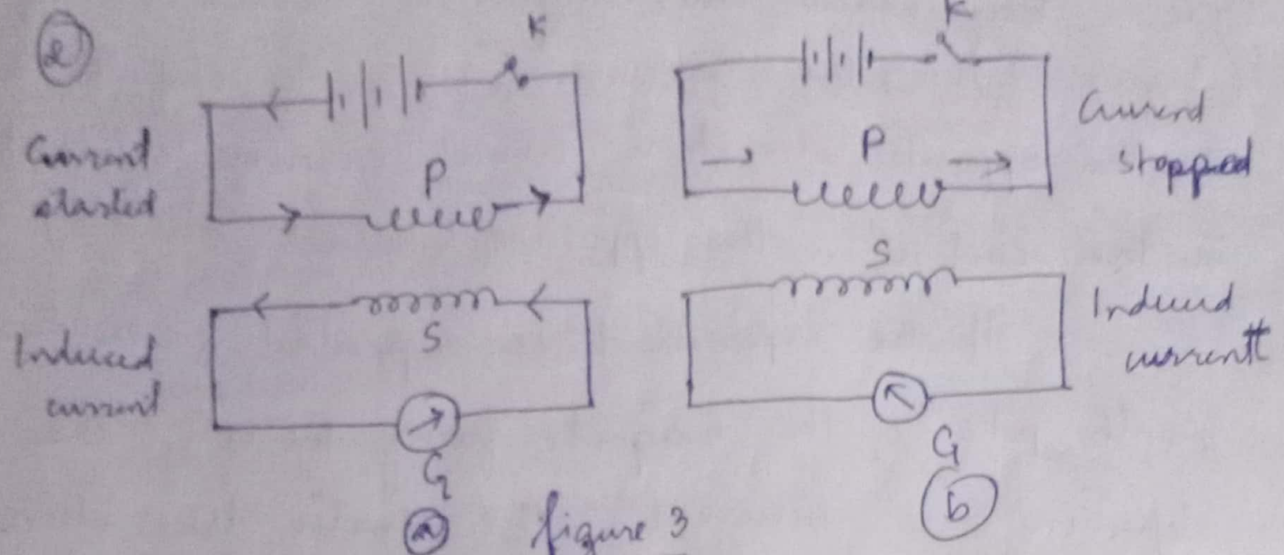


figure 3

Figure shows a primary coil P connected to a battery, and a secondary coil S connected to a galvanometer. When a current in primary coil is started by closing the key K, the galvanometer deflects momentarily, showing an induced current in the secondary in a direction opposite to that in the primary. Similarly, when the current in primary is stopped (fig b), there is again a momentary current in the secondary, but in the same direction as primary. Similar effects are observed while increasing or decreasing the primary current / changing the relative position of the coils. When the coils are wound on a piece of iron, the strength of induced current is much more increased. The induced current is increased still more when the coils are wound on the same closed iron ring.

Faraday's laws of Electromagnetic Induction

First law:

When the magnetic flux through a circuit is changing, an induced emf is set up in the circuit whose magnitude is equal to the negative rate of change of magnetic flux. —— Neumann's law.

If $\Delta\phi$ be the change in magnetic flux in a time interval Δt, then the emf induced in the circuit is given by,

$$\boxed{e = -\frac{\Delta\phi}{\Delta t}}$$

In the limit,

$$\Delta t \to 0,$$

$$e = -\frac{d\phi}{dt} \qquad \text{weber/sec} = \text{volt}.$$

2nd law: The direction of the induced emf or current is such as to oppose the change that produced it.

Explanation: In the magnet and coil experiment, the direction of the induced current is in accordance with ~~Lent~~ Lenz law, that is it opposes the motion of the magnet which produces it. When N pole of the magnet is moved towards the coil, the induced current flows in a direction so that the near (left) face of the coil acts as a magnetic N-pole. The repulsion between the two poles opposes the motion of the magnet towards the coil.

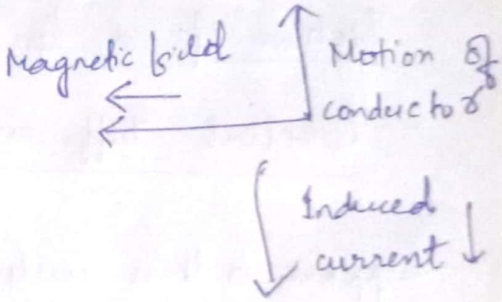
Similarly, when the N-pole of the magnet is moved away from the coil, the direction of the induced current is such as to make the near face of the coil a south pole. The attraction between the two poles opposes the motion of the magnet away from the coil. In either case, therefore work has to be done in moving the magnet.

In either case, work has to be done in moving the magnet. It is this mechanical work which appears as electrical energy in the coil.

If the direction of the induced current were such as not to oppose the motion of the magnet, then we would be obtaining electrical energy continuously without doing any work, which is imposs-ble. Thus Lenz law is in accordance with the principle of conservation of energy.

Direction of Induced current: Fleming's Right hand rule:

If we stretch the R-H thumb and two nearby fingers perpendicular to one another, and the first finger points in the direction of magnetic field and the thumb in the direction of motion of the conductor, then the middle finger will point in the direction of the induced current.



Induced Current and Induced Charge:

If in a coil of N turns the rate of change of magnetic flux be $\Delta\phi/\Delta t$, then the induced emf in the circuit is —

$$\boxed{e = -N\frac{\Delta\phi}{\Delta t}}$$ Faraday's law.

If the coil be closed and the total resistance of its circuit be R, then the induced current in the circuit will be —

$$i = \frac{e}{R} = \frac{N}{R}\frac{\Delta\phi}{\Delta t}$$

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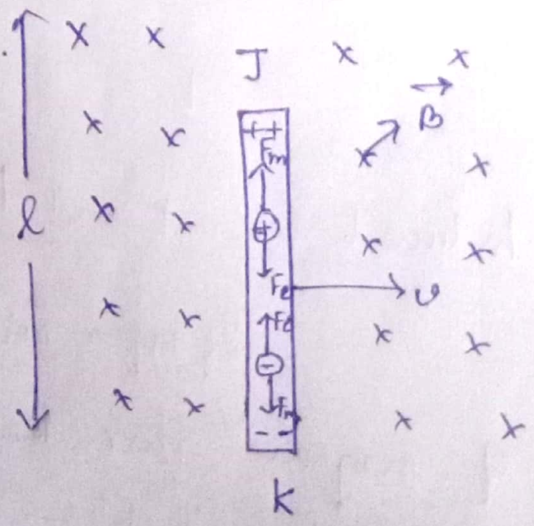
It is clear from this equation that the induced current in the circuit depends upon the resistance (whereas the induced emf is independent of resistance).
The charge flowed through the circuit in a time-interval Δt will be given by —

$$q = i \times \Delta t = \frac{N}{R} \frac{\Delta \phi}{\Delta t} \times \Delta t$$

$$q = \frac{N}{R} \Delta \phi = \frac{\text{Number of turns} \times \text{change in magnetic flux}}{\text{Resistance}}$$

Motion of a Conductor in a Magnetic field : Induced Potential Difference



Suppose a thin conducting-rod JK of length l is situated in a uniform magnetic field of magnitude $\vec{B}$ perpendicular to the plane of paper ~~perpendicular~~ directed downward. $\vec{B}$ has been represented by crosses (x). Suppose the rod is moved in the plane of paper perpendicular to its own axis, with a constant velocity v towards right.

We know that there are always some free electrons (negative charge) in a conductor. The atoms from which these electrons are detached have excess of positive charge. Thus, the conductor has some positive and some negative charges. Of these only the negative charges can move about within the conductor. With the conducting-rod moving with velocity v, these positive and negative charges also move in the magnetic field B towards right along with the rod.

We have read that when a charge q moves in a magnetic field B with a velocity v $\perp^r$ to the field, it is acted upon by a magnetic force F_m, where,

$$\boxed{F_m = qvB}$$

'F_m' is called 'Lorentz force' and its direction is perpendicular to both B and v. According to the Fleming's LHR, the force $\vec{F_m}$ on the positive charge will be directed towards $\vec{J}$ and on the negative charge towards K. Since only electrons are free to move within the rod, they start moving towards end K of the rod. Thus there becomes a deficiency of electrons near the end J of the rod and this end becomes positively charged.

Simultaneously, there becomes an excess of electrons at the end K and this end becomes negatively -charged. Hence, an electric potential difference V is induced between the ends of the rod, due to which an electric field E is created within the rod,

where,

$$\boxed{E = V/l} \longrightarrow ①$$

The direction of E is from J to K. The field E imposes an electric force Fe on each charge (q) within the rod, where.

$$\boxed{F_e = qE} \longrightarrow ②$$

The direction of Fe on positive charge will be towards K and on negative charge ~~todars~~ towards J. Thus, the direction of electric force Fe on each charge is opposite to that of the magnetic force Fm. As more and more electrons reach the end K, ~~the~~ magnitude of Fe goes on increasing until it equals Fm. Now, the resultant force on each charge in the rod becomes zero and the movement of electrons towards K stops.

In this condition,

$$F_e = F_m$$

$$qE = qvB \Rightarrow \boxed{E = vB} \longrightarrow ③$$

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from eqⁿ ①, ② & ③

$$\frac{V}{l} = vB$$

$$\Rightarrow \boxed{V = vBl}$$

if v be in meter/second, B in weber/meter^2 and l in meter, then V will be in volt.

Self Induction :

When a current flows through a coil, it produces a magnetic field and hence a magnetic flux which is linked with the coil. If the current through the coil is changed, the flux linked with the coil also changes. Therefore, an induced e.m.f is set up in the coil and an induced current flows through it, besides the main current. According to Lenz's law, the induced current always opposes the change in the main current. When the main current is increased, the induced current flows opposite to the main current and opposes the increase in main current. When the main current is decreased, the induced current flows in the same direction as the main current and opposes the decrease in the main current.

* The phenomenon of EM induction in which, on changing the current in a coil, an opposing induced emf is set up in that very coil is called self induction.

* When the current in a coil is switched on, the self-induction opposes the growth of the current, and when the current is switched off, the self induction opposes the decay of the current. This is why self-induction is called "Inertia" of electricity.

Coefficient of Self-Induction

Let us consider a coil of 'N' turns carrying a current 'i'. Let ϕ be the magnetic flux linked with each turn of the coil. Then the number of flux linkages is $N\phi$. If no magnetic materials (iron etc) are present near the coil, then the number of flux-linkages with the coil is proportional to the current 'i'.

i.e, $N\phi \propto i$

$$N\phi = Li$$

where L is a constant called "coeff of self-induction" or self inductance of the coil

$$\boxed{L = \frac{N\phi}{i}}$$

if $i = 1$, $L = N\phi$.

The coefficient of self-induction of a coil is equal to the number of flux linkages with the coil when unit current is flowing through the coil.

If on changing the current through the coil, the bar induced emf, 'e' in the coil, then by Faraday's law, we have,

$$e = -N\frac{\Delta\phi}{\Delta t} = -\frac{\Delta(N\phi)}{\Delta t}.$$

where $\Delta(N\phi)/\Delta t$ is the rate of change of flux in the coil. But $N\phi = Li$.

$$\therefore e = -\frac{\Delta(Li)}{\Delta t} = -L\frac{\Delta i}{\Delta t}.$$

where, $\Delta i/\Delta t$ is the rate of change of current in the coil. The negative sign indicates that the induced emf e is always in such a direction that it opposes the change of current in the coil. We have,

$$\boxed{L = -\frac{e}{\Delta i/\Delta t}}$$

SI unit of coefficient of self-induction, is

Henry.

Thus 1 Henry is the self-inductance of a coil when an induced emf of 1 volt is set up in the coil due to a current changing at the rate of 1 ampere per second in the coil.

i.e, $1 \text{ Henry} = \frac{1 \text{ Volt}}{1 \text{ ampere / second}}$

Again from

$L = \frac{N\phi}{i} \Rightarrow 1 \text{ Henry} = 1 \text{ weber}/\text{ampere}$

$= 1 \text{ volt - sec}/\text{ampere}$

The smaller units for L are millihenry (mH) and microhenry (μH).

$1 \text{ mH} = 10^{-3} \text{ H}$

$1 \mu\text{H} = 10^{-6} \text{ H}.$

Dimensions of L:

Numerically, the self-inductance is given by

$L = e / \Delta i/\Delta t$

$$L = \frac{e}{\Delta i/\Delta t} = \frac{e\Delta t}{\Delta i}$$

$$\therefore \text{ unit of } L = \frac{\text{volt-second}}{\text{ampere}} = \frac{(\text{Joule}/\text{coulomb})\text{second}}{\text{ampere}}$$

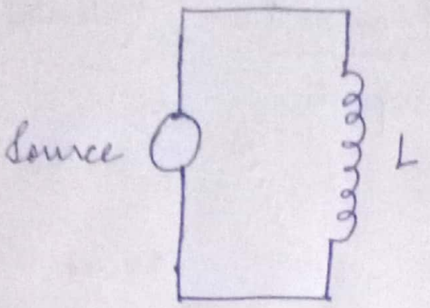
$$= \frac{\text{Newton-meter-second}}{\text{Coulomb-ampere}}$$

$$= \frac{\text{Newton-meter}}{\text{ampere}^2}$$

$$[L] = \frac{[MLT^{-2}][L]}{[A^2]} = [ML^2T^{-2}A^{-2}]$$

Physical significance of Self-Inductance

The self-inductance plays the same role in an electrical circuit as the mass / moment of inertia in mechanical motion. When the circuit is on the self-inductance by inducing an emf slows down the growth of the current. Similarly, when circuit is broken an induced emf in the same direction as original emf thus slows down the decay of the current. If we wish to change the current in a coil, we must overcome this inertia by connecting the coil to some external voltage source such as battery or ~~galvanometer~~ generator as in figure.



In such circuit, the current i depends on the voltage V according to the relation,

$$V = L\frac{di}{dt}$$

Calculation of Self inductance:

① **Solenoid**:

Let l be the length of a long solenoid with an air core and total number of turns are N. When a current 'i' flows through it, the magnetic field inside it is given by equation,

$$B = \mu_0 \frac{Ni}{l} \quad ; \mu_0 \rightarrow \text{permeability constant}$$

Now if A be the area of each turn, then magnetic flux through each turn,

$$\phi_B = \mu_0 \frac{NiA}{l}.$$

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Total flux through the solenoid,

$$= \mu_0 \frac{N i}{l} A N$$

or, $\phi_B = \mu_0 \frac{N^2}{l} A i$

when the current 'i' varies, the flux ϕ_B changes giving rise to the induced emf.

$$\mathcal{E} = -\frac{d}{dt}\left(\mu_0 \frac{N^2}{l} A i\right)$$

$$= -\frac{\mu_0 N^2 A}{l} \frac{di}{dt}$$

$$\mathcal{E} = -L \frac{di}{dt} \qquad \cancel{\left[L = \mu_0 N \right.}$$

$$\therefore L = \frac{\mu_0 N^2 A}{l}.$$

$$\Rightarrow \boxed{L = \mu_0 n^2 l A} \quad ; \quad N = nl.$$

'n' are the number of turns per unit length.

If the solenoid is wound over a core of constant permeability, μ, then, $\boxed{L = \frac{\mu N^2 A}{l}}$ $\quad \mu = \mu_0 \mu_r$

$\mu_r \to$ relative permeability