## 3.1 POTENTIAL DIFFERENCE

Consider an isolated point charge $+q$ lying at $O$ (Fig. 3.1). $A$ and $B$ are two points in its electric field. Let $W_{AB}$ be the work done by an external agent in moving a unit positive charge from $A$ to $B$. *We may define the potential difference between two points in an electric field as the amount of work done in moving a unit positive charge from one point to the other against electrical forces*, or $W_{AB} = V_B - V_A$.

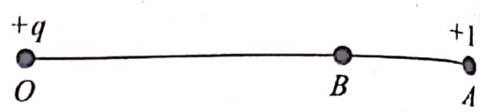

FIG. 3.1

Here, $V_A$ and $V_B$ stand for the potentials at $A$ and $B$.

The SI unit of potential difference is *Volt.*

*The potential difference between two points* is *1 volt if 1 joule of work is done in moving 1 coulomb of charge from one point to the other against electrical forces.*

**Electric Potential.** If $A$ is at infinity, then $V_A = 0$.

$$\therefore \qquad W = V_B$$

Here, $W$ is the work done in moving a unit positive charge from infinity to the point $B$. $V_B$ is the potential at $B$.

Hence the *electrical potential at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point, without acceleration, against electrical forces.*

The potential at a point near an *isolated positive charge is positive.* The potential at a point near an *isolated negative charge is negative.*

**Equipotential Surface.** If all the points of a surface are at the same electric potential, then the surface is called an "equipotential surface".

(*i*) In the case of an isolated point charge, all points equidistant from the charge are at the same potential. Thus equipotential surfaces in this case will be a series of concentric spheres with the point charge as their centre. The potential will, however, be different for different spheres (Fig. 3.2).

(*ii*) For a uniform electric field represented by equidistant parallel lines, the equipotential surfaces are $D$, $E$ and $F$ (Fig. 3.3).

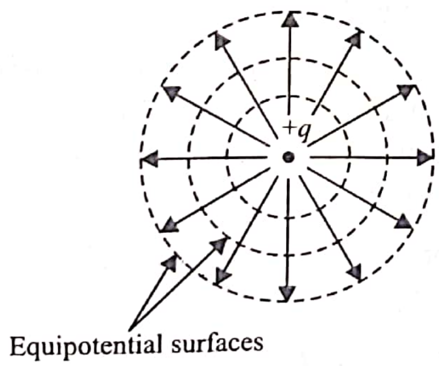

FIG. 3.2

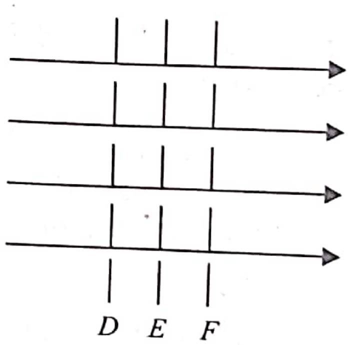

FIG. 3.3

## 3.2 ELECTRIC POTENTIAL AS LINE INTEGRAL OF ELECTRIC FIELD

Let $A$ and $B$ be two points in a non-uniform electric field (Fig. 3.4). Let an external agent move a test charge $q$ from $A$ to $B$ along *any* path. Let **E** be the electric field at any point $P$. The electric field exerts a force $q\mathbf{E}$ on the charge $q$. The external agent must apply a force $F = -q\mathbf{E}$ in order to move $q$ without acceleration.

The work done for a small displacement $d\mathbf{l}$ along $AB = \mathbf{F} \cdot d\mathbf{l}$

$\therefore$ total work done in moving the charge from $A$ to $B$ is

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{l} = -q\int_A^B \mathbf{E} \cdot d\mathbf{l} \text{ (Substituting } \mathbf{F} = -q\mathbf{E})$$

or

$$\frac{W_{AB}}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

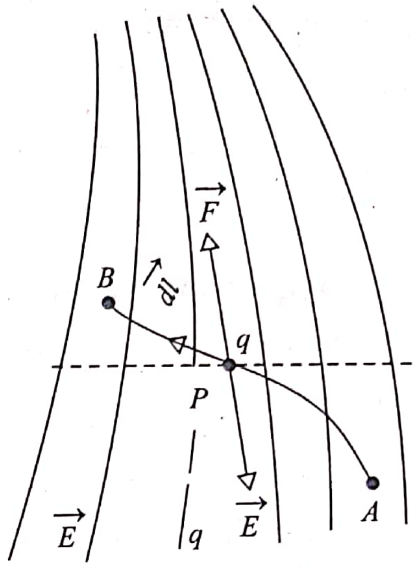


FIG. 3.4

But, by definition, $W_{AB}/q$ is the potential difference $V_B - V_A$ between the points $A$ and $B$. Thus

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

If the point $A$ lies at infinity, $V_A = 0$. Then the potential at the point $B$ is

$$V_B = -\int_\infty^B \mathbf{E} \cdot d\mathbf{l}$$

## 3.3 POTENTIAL AT A POINT DUE TO A POINT CHARGE

Let $+q$ be an isolated point-charge situated in air. $P$ is a point distant $r$ from $+q$ (Fig. 3.5).

The magnitude of the electric field at a distance $r$ from the charge $+q = E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

The potential at $P$ is given by

$$V = -\int_\infty^r \mathbf{E} \cdot d\mathbf{l} \qquad ...(1)$$

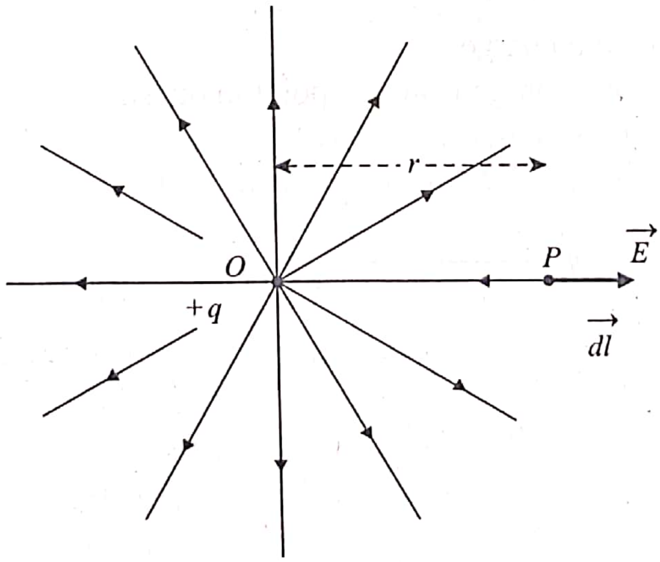


FIG. 3.5

The displacement $d\mathbf{l}$ of the unit charge is directed towards the left. **E** is directed towards the right. Thus the angle between **E** and $d\mathbf{l}$ is 180°.

$\therefore$ $$\mathbf{E} \cdot d\mathbf{l} = E\, dl \cos 180° = -E\, dl$$

$r$ is measured from the charge $+q$ as origin. As we move a distance $d\mathbf{l}$ to the left, the value of $r$ decreases. Thus $dl = -dr$.

$\therefore$ $$\mathbf{E} \cdot d\mathbf{l} = -E\, dl = E\, dr$$

Eq. (1) becomes

$$V = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{l} = -\int_{\infty}^{r} E\, dr = -\frac{q}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{dr}{r^2}$$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

**Potential Difference between Two Points.** Let $r_A$ and $r_B$ be the distances of two points $A$ and $B$ from the charge $+q$. Potential difference between $A$ and $B$ is given by

$$V_B - V_A = -\frac{q}{4\pi\varepsilon_0} \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0} \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$

## 3.4 RELATION BETWEEN ELECTRIC FIELD AND ELECTRIC POTENTIAL

We can calculate the electric field **E** if potential function $V$ is known throughout a certain region of space. Consider two neighbouring points $A$ $(x, y, z)$ and $B$ $(x + dx, y + dy, z + dz)$ distance $d\mathbf{l}$ apart in the region (Fig. 3.9). Let the value of potential at $A$ and $B$ be $V$ and $V + dV$ respectively. Let $dV$ be the change in potential in going from $A$ to $B$. Then

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left( \mathbf{i}\frac{\partial V}{\partial x} + \mathbf{j}\frac{\partial V}{\partial y} + \mathbf{k}\frac{\partial V}{\partial z} \right) \cdot$$

$$(\mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz)$$

Here, $\mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz$ is the displacement vector $d\mathbf{l}$ between $A$ and $B$.

Thus $$dV = (grad\ V) \cdot d\mathbf{l} \qquad ...(1)$$

The work done by the external agent in moving a test charge $q$ from $A$ to $B$ along $d\mathbf{l}$ is

$$dW = \mathbf{F} \cdot d\mathbf{l} = -q\mathbf{E} \cdot d\mathbf{l}.$$

or $$dW/q = -\mathbf{E} \cdot d\mathbf{l}$$

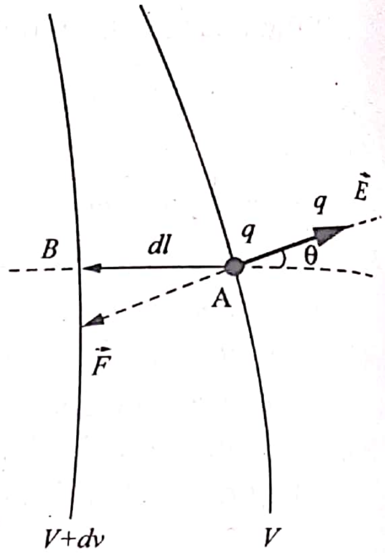

FIG. 3.9

But, by definition, $dW/q$ is the potential difference $dV$ between the points $A$ and $B$. Thus

$$dV = -\mathbf{E} \cdot d\mathbf{l} \qquad ...(2)$$

Comparing Eqs. (1) and (2),

$$\mathbf{E} = -\text{ grad } V = -\nabla V \qquad ...(3)$$

Thus the electric field at any point is the negative of the gradient of potential at that point. The minus sign indicates that $E$ points in the direction of decreasing $V$.

Let $E_x$, $E_y$ and $E_z$ be the components of $\mathbf{E}$ along $x$, $y$ and $z$ axes. Then

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}.$$