2nd & 4th Combined Class
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Topic : CRAMER'S RULE
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The technique of finding Inverse of a Matrix & may be stated in the following steps:

Step I: Find the Determinant, $|A|$

Step II: Find the Co-factor Matrix, Cofactor (A)

Step III: Find the Adjoint Matrix, Adj (A)

Now, since the Matrix Format is

$$AX = C$$

$$\therefore \quad X = \frac{C}{A}$$

$$\text{or} \quad \cancel{X}\ X = C^{-}A$$

$$\text{or} \quad \boxed{X = A^{-1}C}$$

Here, A to the power -1 is called A Inverse.
Now, the ~~for~~ formula for A^{-1} is

$$A^{-1} = \frac{Adj(A)}{|A|}$$

(Adjoint (A) divided by Determinant (A))

$$\text{or} \quad \boxed{A^{-1} = \frac{1}{|A|} Adj(A)}$$

Remember, this is just the formula to find ONLY A^{-1} (A Inverse). Now, place the formula in Step IV.

Step IV : Find A^{-1} as

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

To be contd...

Let us now take a numerical example of a Matrix A of 3×3 dimension.

$$\text{Matrix } A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{bmatrix}$$

We now go to Step I to find the determinant value of Matrix A
The determinant value is given by

$$\begin{bmatrix} 3 & 2 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{bmatrix} \quad |A| = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{bmatrix} - \left\{ \begin{matrix} 3 & 2 & 2 \\ 2 & & 4 \\ 1 & 3 & 5 \end{matrix} \quad \begin{matrix} 3 & 2 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{matrix} \right\}$$

or $3(5-12) - 2(10-4) + 2(6-1)$

or $3(-7) - 2(6) + 2(5)$

or $-21 - 12 + 10$

or $-33 + 10$

$\therefore |A| = -23$

Now, Let's find Cofactor A.
(Similar to Determinant, we have to find Cofactor A with slight changes. Check how it is formed.)

$$\text{Cofactor } A = \begin{bmatrix} \begin{vmatrix} 1 & 4 \\ 3 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ -\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} \end{bmatrix}$$

$$\text{Co factor } A = \begin{bmatrix} (5-12) & -(10-4) & (6-1) \\ -(10-6) & (15-2) & -(9-2) \\ (8-2) & -(12-4) & (3-4) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -6 & 5 \\ -4 & 13 & -7 \\ 6 & -8 & -1 \end{bmatrix}$$

Having found the Cofactor A, we will now move to step III to find Adj (A)

$$\text{Adj } A = \begin{bmatrix} -7 & -4 & 6 \\ -6 & 13 & -8 \\ 5 & -7 & -1 \end{bmatrix}$$

We know

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$= \frac{1}{-23} \begin{bmatrix} -7 & -4 & 6 \\ -6 & 13 & -8 \\ 5 & -7 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-7}{-23} & \frac{-4}{-23} & \frac{6}{-23} \\ \frac{-6}{-23} & \frac{13}{-23} & \frac{-8}{-23} \\ \frac{5}{-23} & \frac{-7}{-23} & \frac{-1}{-23} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 7/23 & 4/23 & -6/23 \\ 6/23 & -13/23 & 8/23 \\ -5/23 & 7/23 & 1/23 \end{bmatrix}$$