3.3 TYPES OF MATRICES

A particular structure of a matrix is normally identified by the name given to a particular matrix. So we can have various types of matrices depending on the dimension of the matrix and the value of its elements.

(I) Square matrix

A matrix whose number of rows and number of columns are equal is called a square matrix. Matrices A and B are square matrix, but C, is not a square matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}; B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3}; C = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times 2}$$

(II) Identify or unit matrix

The square matrix whose principal diagonal elements are 1 and all other elements are zero, is called an identity matrix or unit matrix. It must be noted that identity must always be a square matrix. We can have identity matrix of order 2 or order 3 or any other order depending on the dimension of square matrix and symbolised by I_2 or I_3 or I_n etc.

$$\therefore \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}; I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}; I_n = \begin{bmatrix} 1 & 0 \ldots\ldots\ldots 0 \\ 0 & 1 \ldots\ldots\ldots 0 \\ \vdots & \\ 0 & 0 \ldots\ldots\ldots 1 \end{bmatrix}_{n\times n}$$

The identity matrix plays a role similar to the role played by number 1 in ordinary algebra. In ordinary algebra, for any number a, $1 \times a = a \times 1 = a$. The same is true in case of identity matrix. Any matrix, say A, pre-multiplied or post-multiplied by an identity matrix will give the original matrix (A). Thus

$$IA = AI = A.$$

For example if $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \end{bmatrix}$, then

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 2\times 1+0\times 1 & 1\times 1+0\times 4 & 1\times 3+0\times 2 \\ 0\times 2+1\times 1 & 0\times 1+1\times 4 & 0\times 3+1\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \end{bmatrix} = A$$

$$AI = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \end{bmatrix}_{2\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 2\times1+1\times0+3\times0 & 2\times0+1\times1+3\times0 & 2\times0+1\times0+3\times1 \\ 1\times1+4\times0+2\times0 & 1\times0+4\times1+2\times0 & 1\times0+4\times0+2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \end{bmatrix} = A$$

Another feature of identity matrix is that we can insert or delete an identity matrix during the process of multiplication without affecting the matrix product. Suppose we have a product of two matrices AB. We can also write $AB = AIB$ where I is identity matrix.

(III) Scalar matrix

A matrix which has a common scalar element in the principal diagonal and zeros everywhere else is known as scalar matrix. If λ is the common scalar, the scalar matrix will be

$$\begin{bmatrix} \lambda & 0 & \dots & 0 \\ 0 & \lambda & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & \lambda \end{bmatrix}_{n\times n} = \lambda I_n$$

A scalar matrix needs be a square matrix.

(IV) Diagonal matrix

A matrix which has scalar elements, not necessarily equal, in the principal diagonal and zeros in the off-diagonal positions is called a diagonal matrix. For instance

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ is a diagonal matrix.}$$

In general term,

$$A = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}_{n\times n} = [a_{ij}] \quad \begin{matrix} i, j = 1, 2, \dots, n \\ a_{ij} = 0 \;\; i \neq j \end{matrix}$$

is a diagonal matrix of order $(n \times n)$. The diagonal matrix needs be a square matrix.

(V) Null matrix

A matrix whose all elements are zero, is defined as a null matrix and is symbolised by capital O. Unlike identity matrix, null matrix need not be a square matrix. For instance, we can have null matrix

$$\underset{(2\times 2)}{O} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2} \text{ or } \underset{(3\times 2)}{O} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$$

or,

$$\underset{(n\times n)}{O} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 0 \end{bmatrix}_{n\times n}$$

The function of null matrix is almost similar to the function of zero in ordinary algebra. Just like in ordinary algebra $a \times 0 = 0$ and $a + 0 = a$, in matrix algebra also

$$A \cdot 0 = 0 \quad \text{and} \quad A + 0 = A.$$

But unlike ordinary algebra, $AB = 0$ does not imply that either A or B is a null matrix.

(VI) Symmetric matrix

A matrix whose elements are such that if we interchange the corresponding rows and columns the matrix remains the same, is called a symmetric matrix. In symmetric matrix, therefore, the elements of corresponding rows and columns are the same.

For instance,

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix} \text{ or } A = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 5 \\ 2 & 5 & 3 \end{bmatrix}$$

represents symmetric matrix. In general term

$$A = [a_{ij}]_{n\times n} \quad \text{where } a_{ij} = a_{ji} \qquad (3.14)$$
$$i, j = 1, 2, \dots n$$

is a symmetric matrix of order $(n \times n)$. It may be noted that identity matrix is a special variety of symmetric matrix.

(VII) Triangular matrix

A matrix whose all the elements in one side of the principal diagonal are zeros, is called a triangular matrix. For instance

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 5 & 2 & 4 & 0 \\ 1 & 3 & 2 & 2 \end{bmatrix}_{4\times 4} \text{ and } B = \begin{bmatrix} 2 & 5 & 1 & 3 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}_{4\times 4}$$

It is called triangular matrix, because both the zero elements and the non-zero elements form a triangle.

There are some other types of matrices like singular matrix, non-singular matrix, orthogonal matrix, idempotent matrix etc., but they will be discussed only when the concepts of transpose of matrix, determinant, inverse matrix etc. are explained in the subsequent sections.