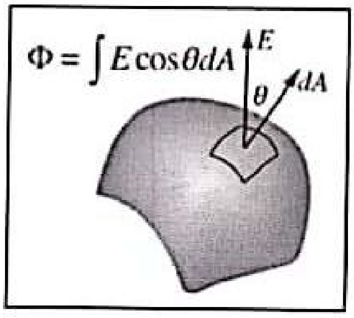
** Electric flux:- It is defined as the total number of lines of force passing normally through any surface placed in the field. It is denoted by ϕ and is a scalar quantity.

It is given by the dot product of $\vec{E}$ and normal infinitesimal area $\vec{dA}$ integrated over a closed surface

$$d\phi = \vec{E} \cdot \vec{dA} = E\, dA \cos\theta$$

$$\Rightarrow \phi = \int E \cos\theta \, dA$$



where θ is the angle between the electric field and normal to the area.

*Note:- (i) SI unit of flux V-m and Nm^2/C

(ii) Dimension: $[ML^3T^{-3}A^{-1}]$

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

(iv) Flux due to the positive charge goes out of the surface while that due to negative charge comes into the surface.

(v) Flux entering a surface is taken as positive while flux leaving is taken as negative.

(vi) The value of electric flux is independent of the shape and size of the closed surface.

(vii) If only a dipole is present within a closed surface then net flux is zero.

(viii) Net flux associated with a closed surface kept in a uniform electric field is zero.

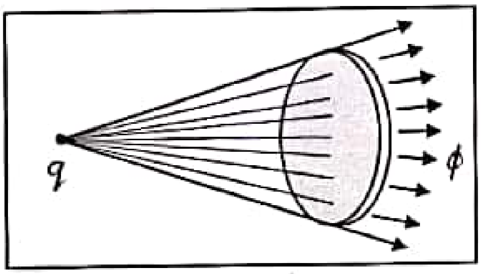
(ix) Net flux from a surface is zero does not imply that intensity of field is also zero.

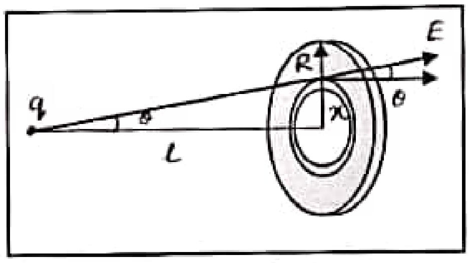
*** (For JEE)

*Electric flux through a circular disc due to a point charge:-

Let us consider a point charge q at a distance l from a disc of radius R. We are to find the electric flux through the disc surface due to the point charge q. The point charge q originates electric field lines radially outward direction. The field lines originated in a cone that passes through the disc surface.

To calculate the flux, let us consider an elemental ring on the disc surface of radius x and width dx as shown in the figure. Area of the ring strip is $dS = 2\pi x\, dx$. The electric field due to the point charge q at this elemental ring is given by $$E = \frac{1}{4\pi\epsilon_0}\frac{q}{(x^2+l^2)} = K\frac{q}{(x^2+l^2)} \longrightarrow ①$$





If $d\phi$ is the flux passing through this elemental ring, then

$$d\phi = E\,dS\cos\theta = \frac{Kq}{(x^2+l^2)}\,2\pi x\,dx\,\frac{l}{\sqrt{x^2+l^2}}$$

$$= \frac{2\pi K q\, l\, x\, dx}{(x^2+l^2)^{3/2}}$$

Total flux

$$\Rightarrow \phi = \int d\phi = \int_0^R \frac{q l\, \cancel{2\pi}}{{}_2\cancel{4\pi}\epsilon_0}\frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \int_0^R \frac{ql}{2\epsilon_0}\frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \frac{ql}{2\epsilon_0}\int_0^R \frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$= \frac{ql}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+l^2}}\right]_0^R$$

$$\Rightarrow \phi = \frac{ql}{2\epsilon_0}\left[\frac{1}{l} - \frac{1}{\sqrt{R^2+l^2}}\right]$$

*** **Gauss law:–** This law states that the electric flux ϕ_E through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

i.e $$\phi_E = \oint \vec{E}\cdot d\vec{s}$$

$$= \frac{q_{enc}}{\epsilon_0}$$

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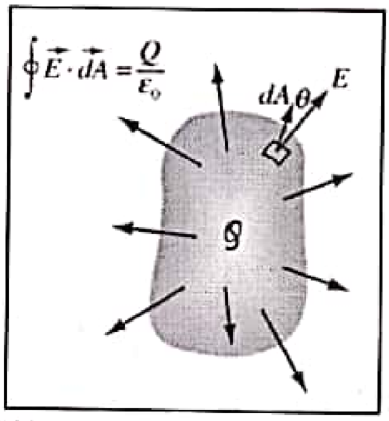
Here the charge enclosed by the surface $q_{enc} = Q$.

The closed surface can be hypothetical and then it is called a Gaussian surface.

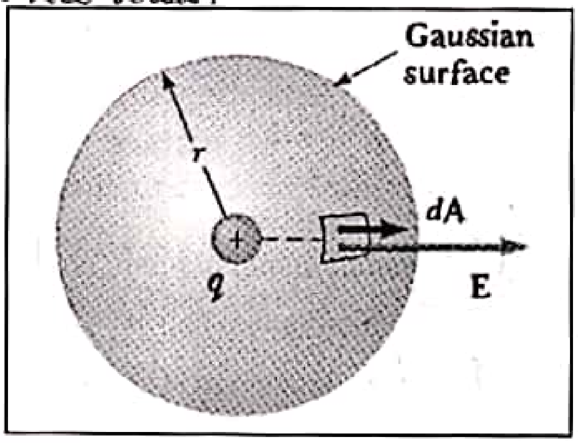


If the closed surface enclosed a number of charges $q_1, q_2 \cdots\cdots q_n$, then

$$\phi = \int \vec{E}\cdot d\vec{s}$$

$$= \frac{(q_1 + q_2 + \cdots\cdots + q_n)}{\epsilon_0}$$

** Gauss's law and Coulomb's law are equivalent, i.e if we assume Coulomb's law we can prove Gauss's law and vice-versa.



To prove Gauss's law from coulomb's law let us consider a gaussian surface of radius r with point charge q at the centre as shown in the figure.

From coulomb's law the electric field intensity at the point P on the surface will be

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q\vec{r}}{|\vec{r}|^3}$$

And the electric flux linked with the area $d\vec{A}$ is

$$d\phi = \vec{E}\cdot d\vec{A} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\vec{r}\cdot d\vec{s}$$

∴ The total flux linked with the whole spherical surface

$$\phi = \int d\phi = \int \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}|^3}\vec{r}\cdot d\vec{s}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\int \hat{r}\cdot d\vec{s}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\int |\hat{r}|\,|d\vec{s}|\cos 0^\circ$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,4\pi r^2$$

$$\phi = \frac{q}{\epsilon_0}$$

Though here we have assumed the surface to be spherical, it is true for any arbitrary closed surface.

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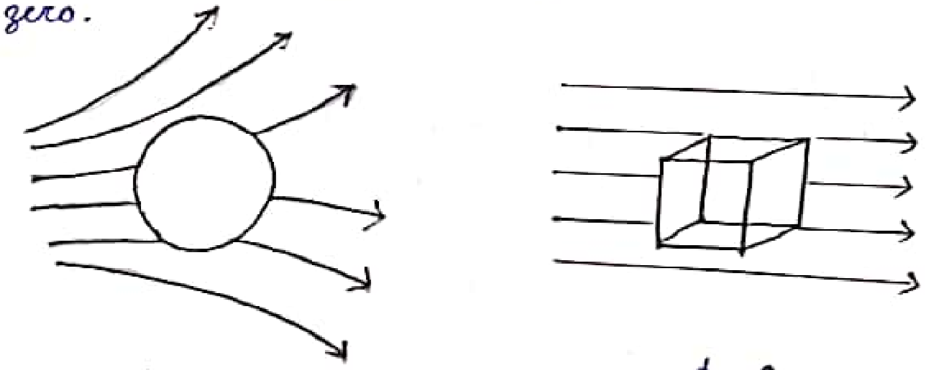
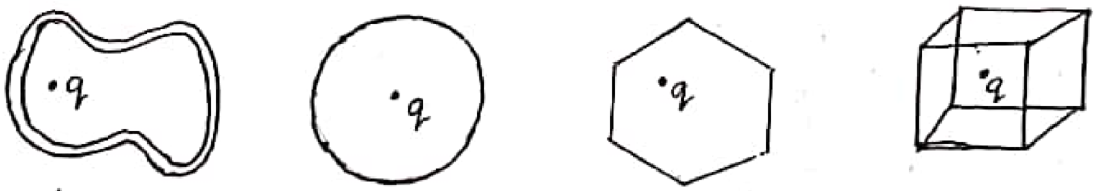
* Note:- If a body (not enclosing any charge) is placed in an electric field either uniform or non uniform total flux linked with the body is will be zero.



$\phi = 0$ $\phi = 0$

*Note:- If the closed body encloses a charge q, then total flux, linked with the body will be

$$\oint_S \vec{E}\cdot\vec{dS} = \frac{q}{\epsilon_0}$$



$\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$ $\phi = q/\epsilon_0$

From the above expression of flux and the figure it is clear that the flux linked with a closed body is independent of the shape and the size of the body and the position of the charge inside the body.

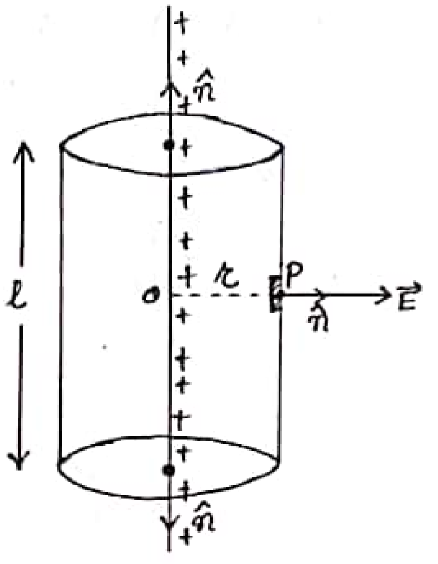
* Note:- For the case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}\phi_E = \frac{q}{2\epsilon_0}$ and if the symmetrical closed body has 'n' identical faces with point charge at its centre flux linked with each face will be $\frac{\phi}{n} = \frac{q}{n\epsilon_0}$.

* Note: Gauss law is a powerful tool for calculating electric field intensity in case of symmetrical charge distribution by choosing a Gaussian surface in such a way that $\vec{E}$ is parallel or perpendicular to its various faces.

** Application of Gauss's law :–

*1. Electric field due to an infinitely long straight uniformly charged wire :-

Let us consider an infinitely long thin wire of uniform linear charge density λ. We are to calculate the electric field intensity at any point P at a distance r from the wire.



Let us consider a cylindrical Gaussian Surface of radius r and length 'l' as shown in the figure. The magnitude of the electric field at each point on the curved surface of the cylinder is same. Let the field at the point P be E.

Here for the curved surface the angle between the field $\vec{E}$ and the area ($\hat{n}$) is $0°$ and for the two plane surface of the cylinder the value of angle is $90°$. Hence only the curved surface contribute to the electric flux.

$\therefore$ Electric flux through the Gaussian surface is

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \int_{curved} \vec{E}\cdot\vec{ds} + \int_{Plane} \vec{E}\cdot\vec{ds} + \int_{Plane} \vec{E}\cdot\vec{ds}$$

$$= \int_{Curved} E\, ds \cos 0° + 2\int_{Plane} E\, ds \cos 90°$$

$$\phi = \int_{Curved} E\, dS = E\int_0^{2\pi r l} dS = 2E\pi r l \longrightarrow ①$$

From the Gauss law we have

$$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \longrightarrow ②$$

$$①, ② \Rightarrow 2E\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\epsilon_0}$$

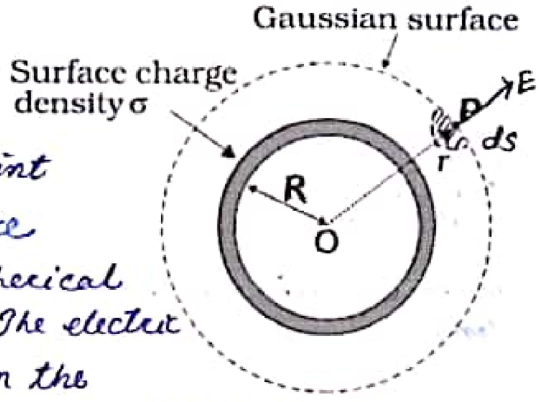
$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow E \propto \frac{1}{r}$$

*2. Electric field due to uniformly charged spherical shell :-

(a) At a point outside the shell.

Let us consider a spherical shell of radius R with centre at O. Let a charge +q be distributed uniformly over the surface of the shell. We are to determine the electric field intensity at any point P at a distance r from the centre of the shell. Let us consider a spherical Gaussian surface of radius 'r'. The electric field intensity at every points on the Gaussian surface is same and say this field be E.



From Gauss's law we have the electric flux through the Gaussian surface

$$\phi = \oint \vec{E}\cdot\vec{ds} = \frac{q_{enc}}{\epsilon_0}$$

Here $q_{enc} = q$
$= \sigma 4\pi R^2$

$$\Rightarrow \oint |\vec{E}||\vec{ds}| \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \oint ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \, 4\pi r^2 = \frac{q}{\epsilon_0}$$

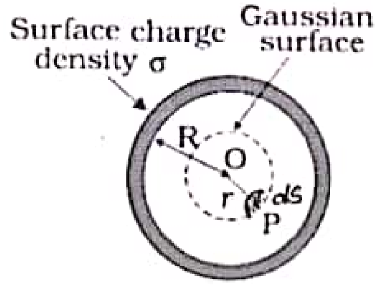
$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

The direction of the field E is radially outward.

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{\sigma 4\pi R^2}{r^2} \longrightarrow (1)$$

(b) At a point inside the shell :-

Let us consider a spherical Gaussian surface through the point P of radius r. We are to determine the electric field intensity at the point P. Here the point P is inside the shell ($\because r < R$)



$\therefore$ From Gauss law,

$$\phi = \oint E\cdot ds = \frac{q_{enc}}{\epsilon_0}$$

Here $q_{enc} = 0$

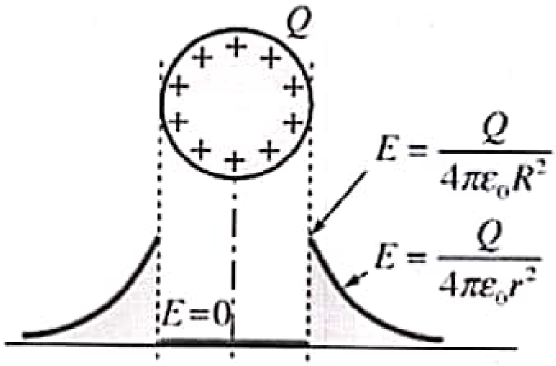
$$\therefore E = 0 \longrightarrow (2)$$

c. At the surface of the spherical shell.

At the surface of the sphere $r = R$

$\therefore$ From eq^n (1)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r} \longrightarrow (3)$$



$\therefore$ From eq^n (1), (2) and (3)

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad r > R$$

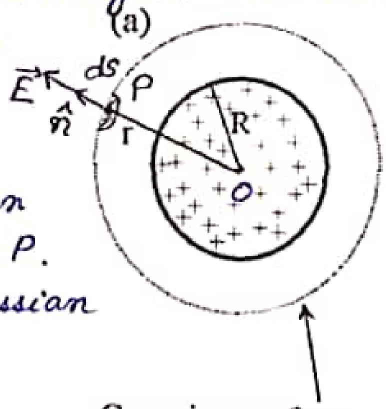
$$E_{in} = 0 \qquad r < R$$

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} \qquad r = R \quad (\text{Maximum value of } E.)$$

* 3. Electric field due to a non-conducting charged solid Sphere:-

a. At a point outside the sphere

Let us consider a non-conducting solid sphere of radius R and centre at O. has uniform charge density ρ. We are to calculate the electric field intensity at a point P outside the sphere at a distance 'r' from the centre of the sphere.



Gaussian surfaces

Let us consider a spherical Gaussian surface of radius 'r' through the point P. Electric field at each point of the Gaussian surface is same and say it is E.

$\therefore$ From Gauss's law

$$\phi = \oint E.ds = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\oint ds = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \rightarrow (1)$$

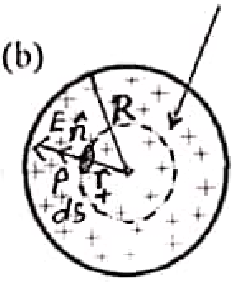
Here angle between $\vec{E}$ and the area $ds\hat{n}$ is 0°. and Q is the total charge enclosed by the Gaussian surface. $Q = \rho\frac{4}{3}\pi R^3$.

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\rho\, \frac{4}{3}\pi R^3}{r^2}\hat{r}$$

Hence $\vec{E}_{out} \propto \frac{1}{r^2}$

(b). At a point inside the sphere:

Let us consider a spherical Gaussian surface of radius r (such that $r < R$) through the point P. We are to find the electric field at the point P.



From the Gauss law we have

$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

Here the electric field E is same at every point on the Gaussian surface. The angle between the field $\vec{E}$ and the area $dS\hat{n}$ is $0°$.

$$\therefore \int E\, dS \cos 0° = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E \int dS = \frac{4}{3} \frac{\pi r^3 \rho}{\epsilon_0}$$

$$\Rightarrow E\, 4\pi r^2 = \frac{4}{3} \frac{\pi r^3 \rho}{\epsilon_0}$$

$$\Rightarrow E = \frac{4}{3} \frac{1}{4\pi\epsilon_0} \frac{\pi r^3 \rho}{r^2}$$

$$\Rightarrow \vec{E} = \frac{r\rho}{3\epsilon_0} \hat{r} \longrightarrow (2)$$

$$\Rightarrow E_{in} \propto r$$

Here $q_{enc} = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$

where $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$\therefore q_{enc} = \rho \frac{4}{3}\pi r^3$

(c) At a point on the surface of the sphere

Here $r = R$.

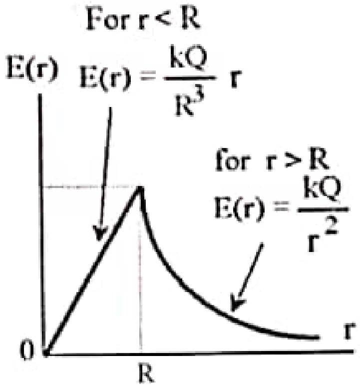
$$\therefore E_s = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{r} \longrightarrow (3)$$

From eq^n (1), (2) and (3) we have

$$E_{out} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \qquad r > R$$

$$E_{in} = \frac{1}{4\pi\epsilon_0} \frac{4}{3} \pi r \rho = \frac{r\rho}{3\epsilon_0} \hat{r} \qquad r < R$$

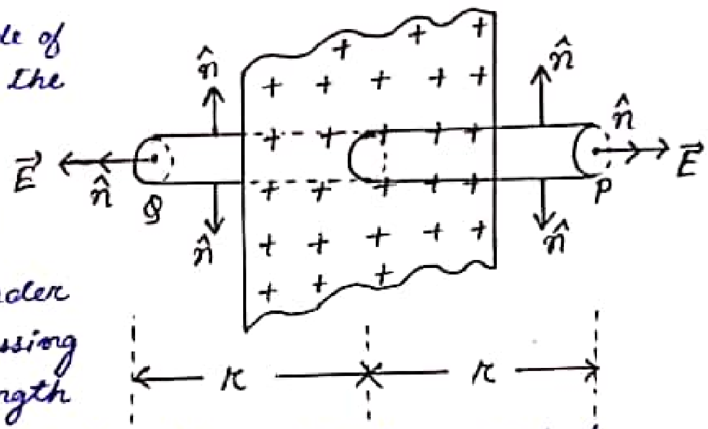
$$E_s = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{r}.$$



* 4. Electric field due to a thin infinite plane sheet of charge:-

Let us consider a thin, infinite plane sheet of charge having charge density σ (surface charge density). We are to determine the electric field intensity at a point P. at a distance r from the sheet.

Electric field on either side of the sheet is perpendicular to the sheet and have the same magnitude at all points equidistant from the sheet.



Let us imagine a cylinder of cross-sectional area dS passing through the sheet having length $2r$. At the two edges of the cylinder the outward normal to the surface is parallel to the field E.

$\therefore$ The electric flux through these edges $= 2\vec{E}.dS\hat{n}$

$= 2EdS$.

The curved surface of the cylinder will not contribute to the electric flux, since for the curved surface the outward normal $\hat{n}$ is perpendicular to the field E.

$\therefore$ The total flux through the cylindrical Gaussian surface is

$$\phi = 2EdS.$$

Now from the Gauss's law we have

$$\phi = 2EdS = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow 2EdS = \frac{\sigma dS}{\epsilon_0}$$

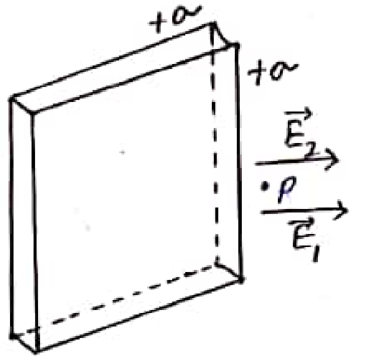
$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Here we have seen that the field E is independent of the distance r.

* Special case:- If the infinite plane sheet has uniform thickness and have uniform surface charge density σ on both the surface of the sheet.

The electric field intensity at any point P due to each surface is $E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$

From the superposition principle we have the net electric field intensity at the point P due to the infinite plane sheet

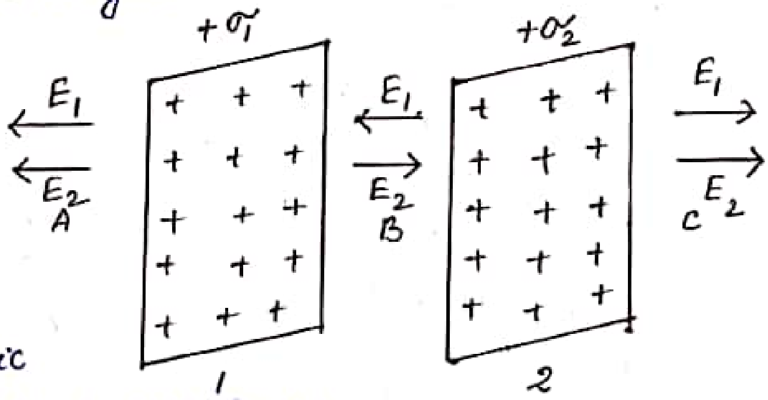


$$E = E_1 + E_2$$
$$= \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$
$$= \frac{\sigma}{\epsilon_0}$$

* Electric field intensity due to two thin infinite parallel sheet of charge.

Let two thin infinite plane sheets of charge density σ_1 and σ_2 are held parallel to each other.

Let E_1 and E_2 are the electric field intensities at a point due to the charged sheet 1 and 2.



Then we have

$$E_1 = \frac{\sigma_1}{2\epsilon_0}$$

and

$$E_2 = \frac{\sigma_2}{2\epsilon_0}$$

From the figure electric field in the region A, B and C are

$$E_A = E_1 + E_2 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

$$E_B = E_1 - E_2 = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$$

$$E_C = E_1 + E_2 = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

Special case: If $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$

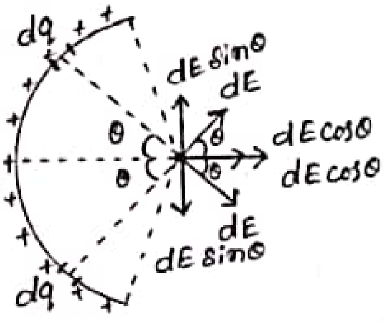
Then

$$E_A = \frac{1}{2\epsilon_0}(\sigma - \sigma) = 0$$

$$E_B = \frac{1}{2\epsilon_0}(\sigma + \sigma) = \frac{\sigma}{\epsilon_0}$$

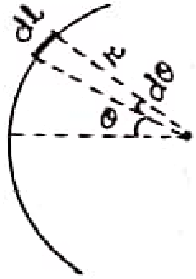
$$E_C = \frac{1}{2\epsilon_0}(\sigma - \sigma) = 0.$$

* Problem :- Linear charge density of an arc is λ and the total angle at the centre is α. Calculate the electric field at the centre.



Let us consider two small elements of the arc of length dl having charge dq as shown in the figure.

If dE be the field at the centre due to the two elements then resolving the two fields as shown in the figure.



From figure we have the net field due to the two elements is

$$dE_N = dE\cos\theta + dE\cos\theta$$

$$= 2\,dE\cos\theta.$$

Again

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dl}{r^2}$$

$$\therefore dE_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda\,dl}{r^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{2\lambda(r\,d\theta)}{r^2}\cos\theta.$$

$$\because dq = \lambda\,dl\,,\quad dl = r\,d\theta$$

$$\Rightarrow dE_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\cos\theta\,d\theta$$

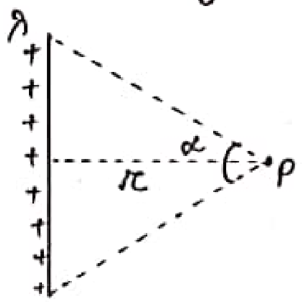
$\therefore$ Total field at the centre is

$$E_N = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\int_0^{\alpha/2}\cos\theta\,d\theta = \frac{2K\lambda}{r}\sin(\alpha/2)$$

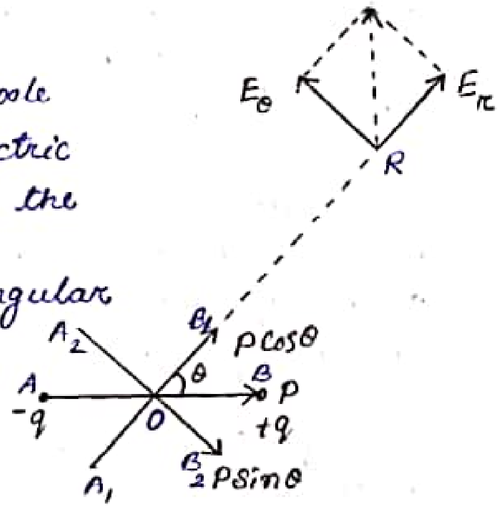
$$\Rightarrow E_N = \frac{2K\lambda}{r}\sin\left(\frac{\alpha}{2}\right)$$

* Note: Electric field due to linear charge distribution :-

$$E_P = \frac{2K\lambda}{r}\sin(\alpha/2)$$



* Electric field due to dipole :-



Let us consider a electric dipole of dipole moment $\vec{P}$. we are to determine the electric field intensity at any point R due to the dipole.

Let us resolve $\vec{P}$ into two rectangular components $p\cos\theta$ and $p\sin\theta$ as shown in the figure.

Electric field intensity at the point R on the axial line of A_1B_1,

$$|\vec{E_r}| = \frac{2(p\cos\theta)}{4\pi\epsilon_0 r^3} \longrightarrow (1)$$

Again, Electric field intensity at the point R on the equitorial line of A_2B_2 is

$$|\vec{E_\theta}| = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \longrightarrow (2)$$

$\therefore$ Net electric field at the point R due to the dipole is,

$$|\vec{E}| = \sqrt{E_r^2 + E_\theta^2 + 2E_r E_\theta \cos 90^\circ}$$

$$= \left[\left\{\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right\}^2 + \left\{\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right\}^2\right]^{1/2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

$$|\vec{E}| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\epsilon_0 r^3} \longrightarrow (3)$$

Special cases :- when $\theta = 0^\circ$ then

$(3) \Rightarrow |\vec{E}| = \frac{2p}{4\pi\epsilon_0 r^3}$ (Field due to dipole on the axial line)

Special case :- when $\theta = 90^\circ$ then

$(3) \Rightarrow |\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}$ (Field due to dipole on the equitorial line)

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