* Electric field intensity :- Force experienced by a unit positive charge placed in an electric field at a point is called the electric field intensity at that point. If q_0 is the test charge placed at any point ($\vec{r}$) and $\vec{F}(r)$ be the force acting on the charge particle then the electric field intensity at the point is given by

$$\vec{E}(\vec{r}) = \frac{\vec{F}(r)}{q_0}$$

* S.I unit of electric field is Newton/coulomb (N/C).
* Electric field intensity is a vector quantity.

As the test charge $+q_0$ may have its own electric field, it may modify the electric field of the source charge. Therefore to minimise this effect and ultimately remove it, we rewrite electric field intensity at $\vec{r}$ as,

$$\vec{E}(\vec{r}) = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

As the magnitude of the test charge decreases, electric field at a point is defined more and more accurately. On account of the descrete nature of charge, the minimum possible value of test charge q_0 is 1.6×10^{-19} C (which is the unit charge). It cannot be zero.

* Dimensional formula of electric field $[M^1 L^1 T^{-3} A^{-1}]$

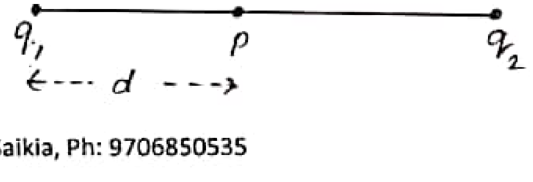
* Direction of electric field due to positive charge is away from charge while direction of electric field due to negative charge is towards the charge.

** Special points to be noted:

a) If q_1 and q_2 are at a distance 'r' and both have the same type of charge, then the distance 'd' of the point from q_1 where electric field is zero is given by

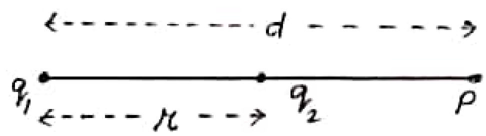
$$d = \frac{\sqrt{q_1}\, r}{\sqrt{q_1} + \sqrt{q_2}}$$

. The point lie between the two charges on the line joining q_1 and q_2.



(b) If q_1 and q_2 have opposite charges then the distance 'd' of the point P from q_1 where electric field is zero is given by

$$d = \frac{\sqrt{q_1}\, r}{\sqrt{q_1} - \sqrt{q_2}} \qquad \text{Here } q_1 > q_2$$



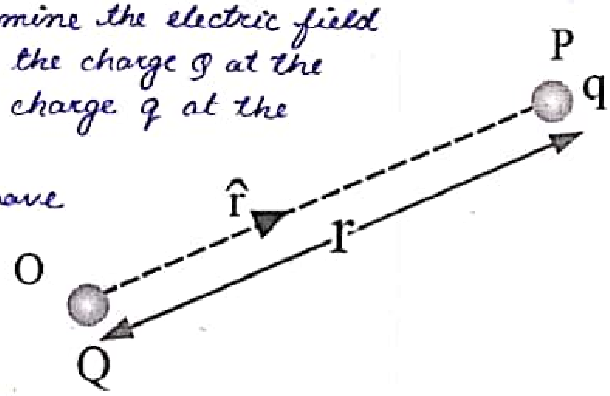
(c) Three charges $+Q_1$, Q_2 and q are placed on a straight line. If these system of charges is in equilibrium, charge q should be as given

$$q = -\frac{Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

** Electric field due to a point charge.

Let us consider a point P at a distance r from the charge Q at point O. We are to determine the electric field intensity at the point P due to the charge Q at the point O. Let us consider a test charge q at the point P.

From Coulomb's law we have the force F between the two point charges is



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector directed from the point O to the point P.

Again we know that the electric field

$$\vec{E} = \frac{\vec{F}}{q}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r}|^3} \vec{r}$$

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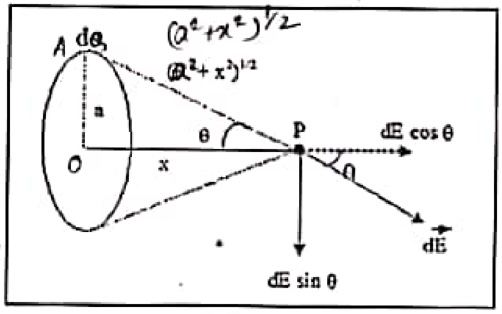
** A ring of radius 'a' contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance of x from the centre of the ring.



Sol^n:-

Let us consider a small element of the ring at the point A having charge dq. The field at the point P due to the element is

$$dE = \frac{dq}{4\pi\epsilon_0 \, AP^2}$$

Resolving the field $d\vec{E}$ along two rectangular components
dE cos θ along the horizontal component
dE sin θ along the vertical component.

Here the vertical components of the field i.e dE sin θ will cancell out. The net field at the point P is only due to the horizontal component.

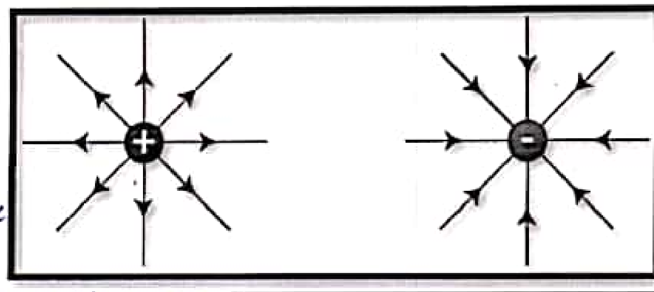
∴ Net field at the point P is $\quad \because \cos\theta = \frac{OP}{AP}$

$$E = \int dE \cos\theta .$$

$$= \int \frac{1}{4\pi\epsilon_0} \frac{dq}{AP^2} \cos\theta = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{(a^2+x^2)^{1/2}} \frac{x}{(a^2+x^2)}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x}{(a^2+x^2)^{3/2}} \int_0^q dq$$

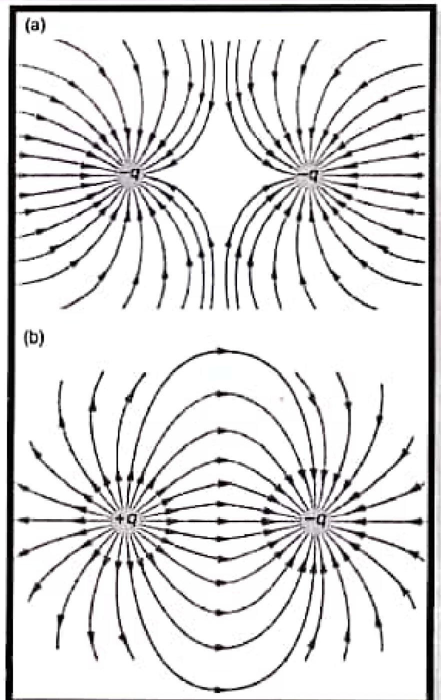
$$E = \frac{1}{4\pi\epsilon_0} \frac{xq}{(a^2+x^2)^{3/2}}$$



** Electric field lines :-

Electric field line in a region can be graphically represented by drawing certain curves known as lines of electric force or electric field lines.

Lines of force are drawn in such a way that the tangent to a line of force gives the direction of resultant field there.



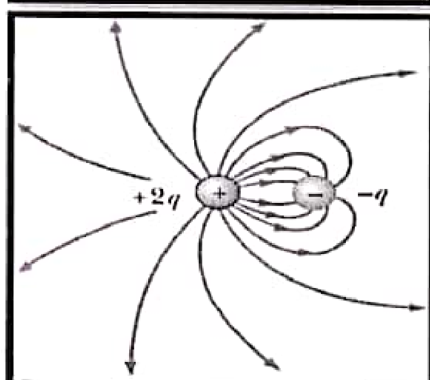
*. Electric field line due to positive point charge is represented by straight lines originating from the charge

* Electric field line due to negative charge is represented by straight lines terminating at the charge

** Properties of electric field lines :-

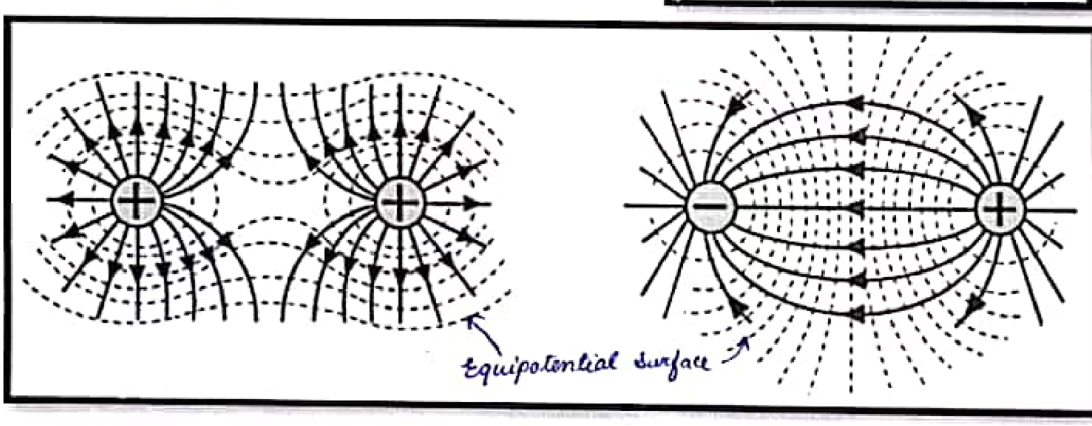
1. Electric lines of force starts from positive charge and end on negative charge.

2. No two lines of force can intersects each other. If they does so then at the point of intersection two tangent could be drawn, which gives two directions of the field at the same point, which is impossible.



3. Tangent drawn at any point on lines of force gives the direction of the force acting on positive charge placed at that point.

4. These lines have a tendency to contract in length as stretched elastic



string. This actually explains the attraction between opposite charges.

5. These lines have tendency to separate from each other in direction perpendicular to their length. This explains repulsion between like charges.

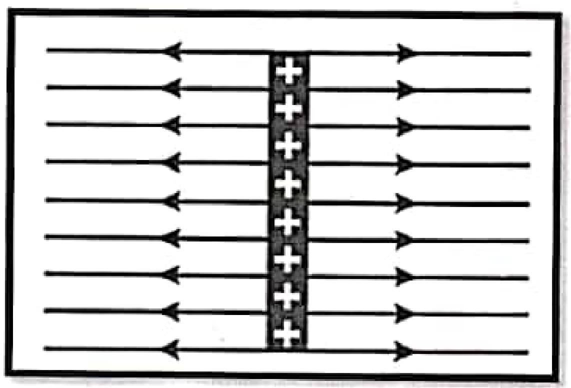
6. Intensity of field line is given by the number of electric field lines in a unit area at that point.

7. Lines of force of a uniform field are parallel and are equally spaced.

8. A unit positive charge gives $\frac{4\pi}{K}$ lines in a medium of dielectric constant K.

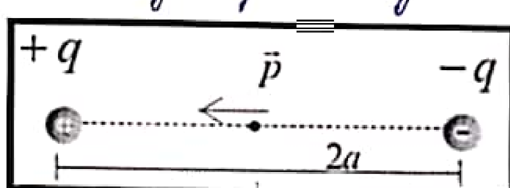
9 Electric lines of force can never enter the conductor, because inside a conductor the intensity of electric field is zero.

10. Lines of force leaves the surface of a conductor normally.



** Electric dipole :-

A system of two equal and opposite charges seperated by a certain distance is called electric dipole, shown in the figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the seperation between the charges.



Dipole moment $\vec{p} = q\vec{d}$ [$d = 2a$ Here]

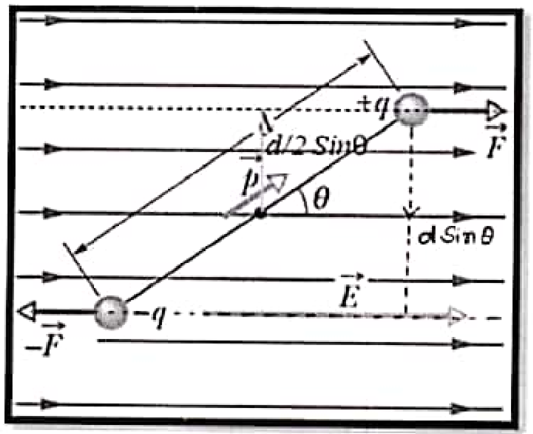
In some molecules the centre of the positive and the negative charges doesn't coincide. This results in formation of dipole. Atom is nonpolar because in it the centre of positive and negative charges concides. Polarity can be in an atom by the application of electric field. Hence it can be called induced dipole.

·Dipole moment is a vector quantity and it is directed from negative to the positive charge.

Dimension of dipole moment $\vec{P} = [L^1 T^1 A^1]$

** Dipole placed in an uniform Electric field :-

Figure shows a dipole of dipole moment $\vec{P}$ placed at an angle θ to the direction of electric field. Here charges of the dipole experience forces qE in the opposite direction.



The net force on the dipole $\vec{F_N} = (qE - q.E) = 0$.

Thus when a dipole is placed in an electric field (uniform) net force on the dipole is zero. But equal and opposite forces acts with a seperation of their line of action, they produce a couple which tends to align the dipole along the direction of the field. The torque due to this couple is given by

$$\tau = \text{Force} \times \text{seperation of lines of action}.$$

$$= qE \ d\sin\theta = E qd \sin\theta = |\vec{E}||\vec{P}| \sin\theta$$

$$\Rightarrow \vec{\tau} = \vec{E} \times \vec{P}$$

** Work done in rotating a dipole in Electric field.

Let us consider a dipole of dipole moment $\vec{P}$ is oriented at an angle θ with the direction of the electric field $\vec{E}$. (Show in the figure above). The torque acting on the dipole, $\vec{\tau}$ is given by

$$\tau = PE \sin\theta.$$

This torque tries to rotate the dipole to align it along $\vec{E}$. Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is

$$dW = \tau \, d\theta. = PE \sin\theta \, d\theta.$$

∴ Total amount of work done in rotating the dipole from orientation θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$= -PE \left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= -PE(\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow W = PE(\cos\theta_1 - \cos\theta_2) \longrightarrow (1)$$

This amount of workdone is equal to the potential energy of the dipole.

$$\therefore U = PE(\cos\theta_1 - \cos\theta_2]$$

* Particular cases;

Case 1:- when the dipole is initially aligned along the electric field. i.e $\theta_1 = 0°$ and we are to set it at angle θ with $\vec{E}$, i.e $\theta_2 = \theta$

$$\therefore W = PE(\cos 0 - \cos\theta)$$

$$= PE(1 - \cos\theta) \qquad [\text{from eq}^n (1)]$$

Case 2:- When the dipole is initially at right angle to $\vec{E}$ ie, $\theta_1 = 90°$, and we have to set it at angle θ with $\vec{E}$ i.e $\theta_2 = \theta$.

$$W = PE(\cos 90° - \cos\theta)$$

$$= -PE\cos\theta$$

** Force on electric dipole in Non-uniform electric field:-

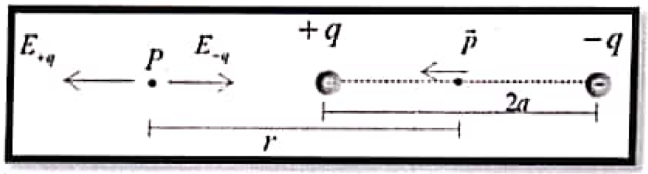
If in a non-uniform electric field a dipole is placed at a point where the field is $\vec{E}$, the interaction energy of dipole at this point is $U = -\vec{P}\cdot\vec{E}$. Now the force on the dipole due to electric field $F = -\frac{\Delta U}{\Delta r}$.

If the dipole is placed in the direction of the field then

$$F = -P\frac{dE}{dx}$$

** Electric field due to a dipole:-

* At a point on the axial line of the dipole:- Let us consider a dipole consisting of two charges $-q$ and $+q$ seperated by a small distance $2a$. Let us consider a point P at a distance r from the centre of the dipole. If E_q be the electric field at the point P due to the charge $+q$ then $|\vec{E}_{+q}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}$ ..... (1)



If E_{-q} be the electric field at the point P due to the charge $-q$ then,

$$|\vec{E}_{-q}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \longrightarrow (2)$$

$\therefore$ Net electric field at the point P is

$$|\vec{E}|^2 = |\vec{E}_{+q}|^2 + |E_{-q}|^2 + 2E_{+q}E_{-q}\cos 180^\circ$$

$$\Rightarrow |\vec{E}| = |\vec{E}_{+q}| - |\vec{E}_{-q}| = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right\}$$

$$\Rightarrow |\vec{E}| = \frac{q}{4\pi\epsilon_0}\frac{4ar}{(r^2-a^2)^2} = \frac{1}{4\pi\epsilon_0}\frac{2r|\vec{p}|}{(r^2-a^2)^2}$$

For a shorter dipole $2a << r$ then

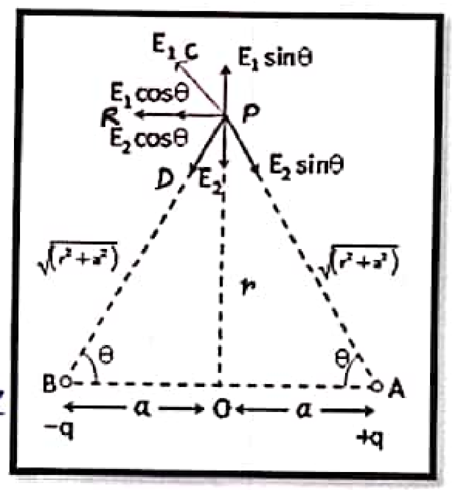
$$\vec{E}_{axial} = \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad \because r^2 - a^2 \approx r^2$$

** At a point on the equatorial line of the dipole :-

Let us consider an electric dipole consisting of two charges $-q$ and $+q$ seperated by a small distance $2a$ with centre at O.

The dipole moment $\vec{p} = q(2a)$

We are to find the electric field intensity at the point P on the equatorial line at a distance of r from the centre of the dipole.



Let E_1 be the field at the point P due to the charge $+q$ at A.

$$\therefore |\vec{E}_1| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \longrightarrow (1) \text{ (along } \vec{PC}\text{)}$$

Similarly electric field at the point P due to the charge $-q$ at the point B

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \longrightarrow ② \text{ (along } \vec{PD}\text{)}$$

$$①, ② \Rightarrow |\vec{E_1}| = |\vec{E_2}| \longrightarrow ③$$

Now resolving the two fields $\vec{E_1}$ and $\vec{E_2}$ along the two rectangular components as shown in the figure.

The two verticle components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and are acting along the opposite direction. Hence they cancel out.

Therefore the net electric field at the point P is only due to the horizontal component. The net electric field at the point P is $\therefore |\vec{E}| = |\vec{E_1}|\cos\theta + |\vec{E_2}|\cos\theta$

$$= 2E_1\cos\theta \qquad \because |\vec{E_1}| = |\vec{E_2}|$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}\cos\theta.$$

$$= \frac{2qa}{4\pi\epsilon_0(r^2+a^2)^{3/2}} \qquad \because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{-\vec{p}}{(r^2+a^2)^{3/2}} \longrightarrow ④$$

The direction of $\vec{E}$ is along $\vec{PR}$ and $\vec{p}$ is opposite to $\vec{E}$.

For a shorter dipole $r >> a \Rightarrow r^2+a^2 \approx r^2$

$$\therefore ④ \Rightarrow |\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

$$\Rightarrow \vec{E}_{equ} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

Note

$$\frac{|\vec{E}_{axial}|}{|\vec{E}_{equ}|} = 2$$