Study Material of B.Sc. III Semester

Electricity and Magnetism

Prepared by: Dr. Tanveer Hasan, Deptt. of Physics, Shia P.G. College, Lucknow, India.

(Reference Book: Electricity and Magnetism by Ahmad & Lal)

Unit-II

Magnetic Force

The force exerted by moving charges or current on each other is called magnetic force. For example force exerted by two parallel current carrying straight wires is magnetic force and it depends upon the direction and magnitude of currents, shape or relative positions of wires.

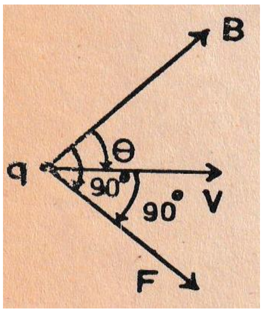


Fig 1

As shown in figure 1, a positive charge q is moving with a velocity 'v' in a magnetic field 'B' then a force 'F' is exerted on it having magnitude:

$$F= qv \times B............. (1)$$

And always directed perpendicular to both, magnetic field and velocity.

Biot-Savart Law

Considering two circuits carrying currents I_1, and I_2, as shown in fig 2. Let us suppose the elements of lengths dl_1, and dl_2 on the two circuits be distant r from each other. It is experimentally found that the force exerted on the current I_1 by current I_2, is given by,

$$\hat{F}= \frac{\mu_0}{4\pi} I_1 I_2 \int_1 \int_2 \frac{dl1 \times (dl2 \times r)}{r^3} .... (2)$$

Where line integrals are calculated over two circuits. μ_0 is the permeability of free space and is numerically equal to $4\pi \times 10^{-7}$ Henry per metre. This is the magnetic force law.

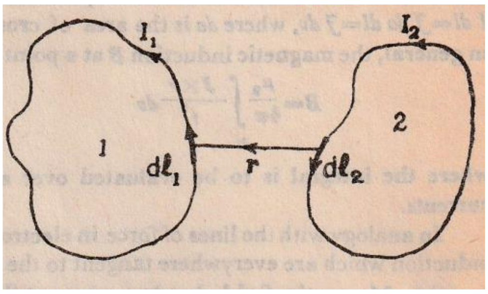


Fig 2

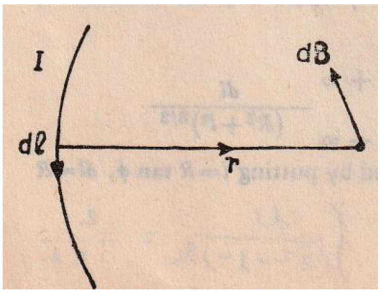


Fig 3

The expression for $F^{\wedge}$ can be rewritten as

$$F^{\wedge} = I_1 \int_1 dl1 \times \frac{\mu_0}{4\pi} I_2 \int_2 \frac{(dl2 \times r)}{r^3}$$

$$= I_1 \int_1 dl1 \times B_2$$

$$\text{Where } B_2 = \frac{\mu_0}{4\pi} I_2 \int_2 \frac{(dl2 \times r)}{r^3} \quad \text{......................(3)}$$

B_2 is the magnetic field due to the circuit 2 at the position of the element dl_1, of circuit l. This is called Biot-Savart law and gives the magnetic field due to a current carrying wire at a distance r from it. Dropping the suffix 2 in eqn. (3) we can write the Biot-Savart law as:

$$B = \frac{\mu_0}{4\pi} I \int \frac{(dl \times r)}{r^3} \quad \text{......................(4)}$$

The vector B, is called magnetic induction or magnetic flux density and is expressed in webers /met. The eqn. (4) can be used to find the magnetic field due to various configurations of current-carrying wire.

From eqn. (4) we have, the magnetic field at a distance r due to an element of length dl of a wire carrying current is :

$$dB = \frac{\mu_0}{4\pi} I \frac{(dl \times r)}{r^3} \quad \text{......................(5)}$$

The vector dB is perpendicular to the plane containing dl and the point of observation as shown in fig (3). From eons (2) and (5) we get, the force on an element of length dl of a wire carrying current I (I dl is called current element) as :

$$dF = Idl \times B \quad \text{......................(6)}$$

if the current is distributed in space having current density $\mathcal{J}$ amp/met^3 then

$$I\, dl = \mathcal{J}\, da\, dl = \mathcal{J}\, dv \quad \text{....................(7);}$$

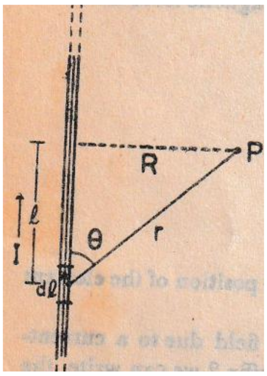
Where da is area of crossection and dv is volume. So in general magnetic induction B can be written as:

$$B = \frac{\mu_0}{4\pi} \int \frac{(\mathcal{J} \times r)}{r^3} dv \quad \text{......................(4)}$$

Applications of Biot-Savart Law

1.Magnetic field due to long straight current carrying wire

Let us calculate the magnetic field B as shown in figure a long straight wire having current 'I'. The magnetic field dB due to an elementary current element 'Idl' at a point P is given by:



Fig

$$dB = \frac{\mu_0}{4\pi} I \frac{(dl\ Sin\theta)}{r^2} \qquad ....(1)$$

so the magnetic field due to whole wire will be given by :

$$B = \frac{\mu_0 I}{4\pi} \int_{l=-\infty}^{+\infty} \frac{(dl\ Sin\theta)}{r^2} \ .... (2);$$

From figure it is clear that: $Sin\theta = \frac{R}{r}$ & $r = \sqrt{(R^2+l^2)}$ .... (3)

Where R is the distance of the wire from point of observation. Form eqn. (3) in eqn. (2)

we have: $$B = \frac{\mu_0 I\, R}{4\pi} \int_{l=-\infty}^{l=+\infty} \frac{dl}{r^3} = B = \frac{\mu_0 I\, R}{4\pi} \int_{l=-\infty}^{l=+\infty} \frac{dl}{\sqrt{(R^2+l^2)}^{\,3}} .... (3);$$

putting; $l = R\tan\phi$ and $dl = R\sec^2\phi\, d\phi$................(4)

we get; $$B = \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{+\pi/2} \cos\phi\, d\phi$$

$$B = \frac{\mu_0}{4\pi}\left(\frac{2I}{R}\right) \ ....(5)$$

The eqn. (5) can be used to define ampere, since $\frac{\mu 0}{4\pi}$ is equal to 10^{-7} M.K.S current through a wire is one ampere when the magnetic field at one metre (R=1m) from it is 2×10^{-7} weber per square meter.

Force between two long parallel wires carrying current

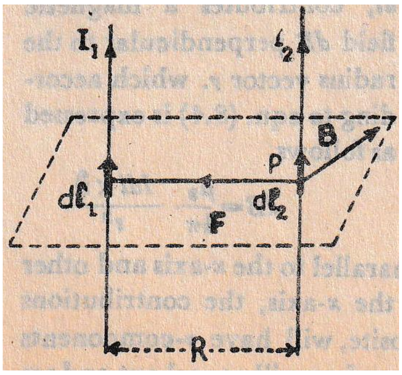
Considering two long parallel wires distant R apart carrying currents I_1 and I_2, say, in the same direction (Fig.1). Let us find the force per unit length experienced by each wire. The magnetic field B at a point P on the second wire due to a current I_1 is given by:

$$B = \frac{\mu_0}{4\pi}\left(\frac{2I1}{R}\right) \ ....(1)$$

and is perpendicular to the plane containing the wire and the point of observation P and will be pointing into the paper as can be seen from right handed screw rule.

The force on an element dl_2 at P due to the field B from I_1, is

$$F = I_2 dl_2 \times B \ ....(2)$$



Fig

Since dl_2 is parallel to dl_1 it will be perpendicular to the field B due to I_1. Therefore we have

$$F = I_2 dl_2\, B \quad ....(3)$$

Now from eqn. (1) in eqn. (3);

$$F = I_2 dl_2 \frac{\mu_0}{4\pi}\left(\frac{2I1}{R}\right) \quad ....(4)$$

Taking $dl_2 = 1$; we have $F = \frac{\mu_0}{4\pi}\left(\frac{2I1I2}{R}\right)$

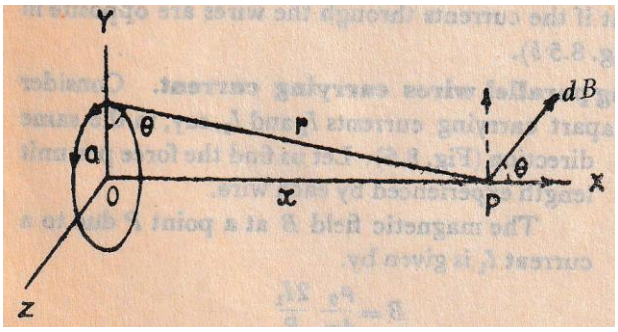
The sign of the force is obtained from cross product. Now if R=1 m, and $I_1 = I_2 = 1$A, then

$$F = 2 \times 10^{-7} \text{ Newton}$$

Definition of 1 Ampere: In M.K.S. one ampere may defined as that current which when flows between two infinitely long straight parallel wires of negligible crossection placed in vacuum one meter apart carrying 1 ampere current each produces a transverse force of magnitude 2×10^{-7} Newton/met between the wires.

Magnetic field along the axis of a circular coil

Consider a circular coil of wire, say lying in the YZ plane, carrying a current I as shown in fig. The magnetic field at an axial point P can be calculated with the help of Biot-Savart law. Each element of the ring of length dl, contributes a magnetic field dB perpendicular to the radius vector $r^{\wedge}$. It is expressed as follows:.



Fig

$$dB = \frac{\mu_0}{4\pi} I \frac{(dl \times r)}{r^3} \quad ....(5)$$

The field dB will have two components, one parallel to the x-axis and other perpendicular to it. As the ring is symmetrical about the x-axis, the contributions dB due to elements of the ring lying diametrically opposite to x axis, will have y-component which is equal in magnitude but opposite in direction, so they all cancel and we are left with x component only.

$$dB = \frac{\mu_0}{4\pi}\frac{(I\,dl)}{r^2}$$

The x-component of dB is given by,

$$dB_x = \frac{\mu_0}{4\pi}\frac{(I\,dl\cos\theta)}{r^2} \quad \text{......................(2)}$$

From figure $\cos\theta = \frac{a}{r}$; where 'a' is the radius of the ring. So eqn (2) becomes;

$$dB_x = \frac{\mu_0}{4\pi}\frac{(I\,dl\,a)}{r^3} \quad \text{...................... (3)}$$

Integrating equation (3) to get magnetic field at due to whole ring P which is at a distance x from the center of the ring;

$$B_x = \oint dBx = \frac{\mu_0}{4\pi}\frac{(I\,a)}{r^3}\oint dl = \frac{\mu_0}{4\pi}\frac{(I\,a)}{r^3}\,2\pi a$$

$$B_x = \frac{\mu_0\,I\,a^2}{2r^3} \quad \text{..........................(4)}$$

Since $r=(a^2 + x^2)^{1/2}$ so $B_x = \frac{\mu_0\,I\,a^2}{2(a^2+x^2)^{3/2}}$;

since dB_x is the only component so dropping suffix;

$$B = \frac{\mu_0\,I\,a^2}{2(a^2+x^2)^{\frac{3}{2}}} \text{.............................(5);}$$

It is clear that magnetic field B varies inversely to the distance 'r' from the center.

At the center of the ring x=0 so;

$$B = \frac{\mu_0\,I\,a^2}{2(a)^3} = \frac{\mu_0\,I}{2a} \text{.............................(6);}$$

If the coil has N turns then field at P will be N times so;

$$B = \frac{\mu_0 N\,I\,a^2}{2(a^2+x^2)^{\frac{3}{2}}} \quad \text{.............................(7);}$$

And at the center $\quad B = \frac{\mu_0\,NI}{2a} \quad$ ....(8);

If point P lies very far from center then x>>a then eqn. (7) becomes;

$$B = \frac{\mu_0 N\,I\,a^2}{2\,x^3} = \frac{\mu_0 N\,I\,a^2}{2\,r^3} \quad \text{.....................(9); (since x=r)}$$

As the area of crossection of the ring is $A=\pi a^2$; so in terms of area we can write as;

$$B = \frac{\mu_0 N\,I\,A}{2\,\pi r^3} \quad \text{.....................(10);}$$

the quantity M=N IA is called magnetic dipole moment so we can write eqn. (10) as follows;

$$B = \frac{\mu_0}{4\pi} \frac{2M}{r^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This formula can be used to calculate the variation of magnetic field due to a Helmholtz galvanometer.

The magnetic due to current carrying solenoid

When a coil of wire is wrapped in the form of a cylinder it is called a solenoid (Fig 1). Fig 2 shows the crossectional view of a solenoid. The path of current in solenoid is helical but if turns are very closely wrapped this can be taken circular rings piled one over the other. Let the number of turns per unit length of solenoid is ‘n’. The magnetic field can be calculated at any point ‘P’ on the axis of the solenoid using eqn. as given below:

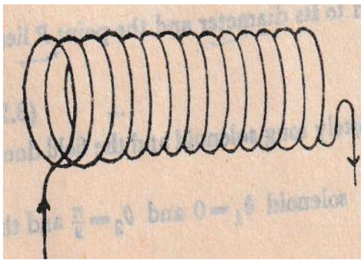


Fig 1

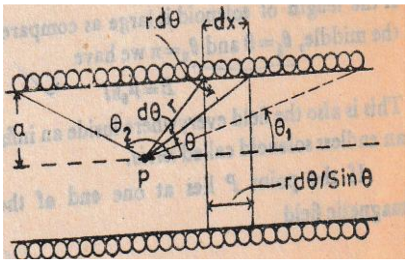


Fig 2

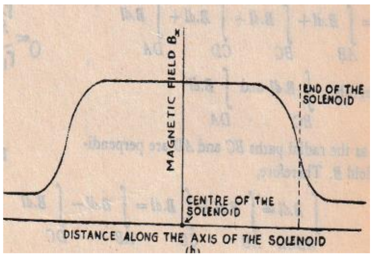


Fig 3

$$B = \frac{\mu_0 I a^2}{2r^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let us first calculate the contribution to the field, from the current rings in a small segment of length dx of the solenoid (fig. 2), which subtends angle θ and (θ+ dθ) with the axis at the point P. From fig 2 we can write:

$$\sin\theta = \frac{r\,d\theta}{dx} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$dx = \frac{r\,d\theta}{\sin\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

the number of turns in the length $dx = ndx = \frac{nr\,d\theta}{\sin\theta}$

and *total current* in the length dx $= I\,\frac{nr\,d\theta}{\sin\theta} \ldots\ldots\ldots\ldots\ldots\ldots (4)$

where I is current in the solenoid $dB = \frac{\mu_0 a^2}{2r^3}\,\frac{I\,nr\,d\theta}{\sin\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$

$$dB = \frac{\mu_0 nI \sin\theta\, d\theta}{2} \ldots\ldots (6) \qquad (\text{since } \frac{a^2}{r^2} = \sin\theta)$$

Integrating it between θ_1 and θ_2 we have,

$$B=\frac{\mu_0 nI}{2}\int_{\theta 1}^{\theta 2} \text{Sin}\theta \, d\theta = \frac{\mu_0 nI}{2}(\cos\theta 1 - \cos\theta 2)..........(7)$$

If the length of the solenoid is large as compared to its diameter then point P lies in the middle and θ_1=0 and θ_2=π then

$$\boldsymbol{B=\mu_0 nI} \text{ ....(8)}$$

This is also the field everywhere inside an infinitely long solenoid and the field due to an endless solenoid called toroid.

If the point P lie at one end of the solenoid θ_1=0 and $\theta_2=\frac{\pi}{2}$ then the magnetic field at P

$$\boldsymbol{B=\frac{\mu_0 nI}{2}}\text{....................(9)}$$

Equations (8) and (9) show, that the magnetic field at the end of a long solenoid is just half to that at center. The graph in fig 3 shows that field at centre of the solenoid remains nearly constant until we approach to one of the ends.